THE INCIDENCE ANGLES OF THE TRACKERS USED FOR THE PV PANELS' ORIENTATION. PART I: EQUATORIAL TRACKERS

Dorin DIACONESCU, Ion VIŞA, Bogdan BURDUHOS, Valentina DINICU
"Transilvania" University of Braşov, Romania

Abstract: This first part of the paper presents the modeling and the simulations of the incidence angle of the (pseudo)equatorial tracker that is the angle between the sunray unit vector and the panel normal unit vector. At the same latitude, a comparison between the tracked PV panel and fixed PV panels is also presented. In order to realize this modelling, in the paper there are modeled the Sun-Earth angles and the two stated unit vectors. The paper's results are useful in the (pseudo)equatorial trackers' design.

Keywords: solar panel, solar (pseudo)equatorial tracker, sunray unit vector, normal unit vector, incidence angle.

1. Introduction

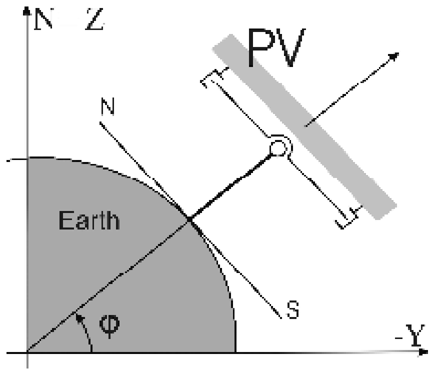
The main objective of this first part of the paper is to model the incidence angle of a PV panel oriented by a (pseudo)equatorial tracker* (see figure 1). In order to achieve this objective, firstly in the paper there are modeled the Sun-Earth angles and the unit vectors of the sunray and of the panel normal.

Secondly the correlation of the incidence angle of the dual-axis (pseudo) equatorial tracker is established and also the simulations of the incidence angle, at 45°N for all three representative days of a year (summer solstice, equinoxes and winter solstice), are presented.

Thirdly and lastly the paper presents the comparative simulations of a horizontal fixed, a tilted fixed and a tracked PV panels and specifies the resulted conclusions.

Fixed PV panels have a very low efficiency degree. In order to increase this coefficient, two possibilities are available. The first option is to use in the design of the panels materials with better solar absorbent properties and the second is to orientate the panels towards the sun, method which is called "Tracking". As the first possibility is quite expensive and new materials take more time to be developed, the second option is more reliable and costs less.

***Remark:** in contrast to the *(pseudo) equatorial tracker* (see figure 1), in the *equatorial tracker* the two revolute joints are assembled in reverse order; for economical reasons the *equatorial tracker* is less used in practice.



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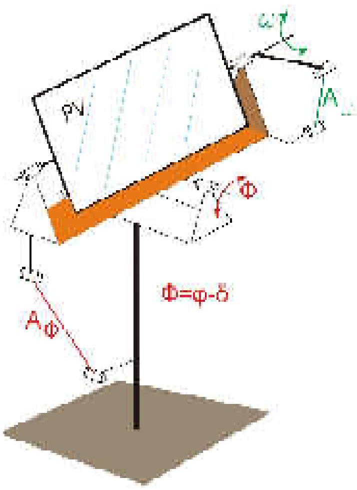

b

Figure 1. (Pseudo)equatorial dual-axis tracker: a) Earth related scheme; b) an operation scheme.

2. Solar Angles and Unit Vectors

In order to be able to track the PV panel one has to know the exact direction of the sun ray. This data can be obtained by using one of the two available systems in which the suns path can be depicted: the *equatorial* (global) system and the *azimuthal* (local) system. The angles used in the first system are (see figure 2): declination (δ) and hour angle (ω), while those used in the second one are (see figures 2 and 4): altitude (α) and azimuth angle (ψ).

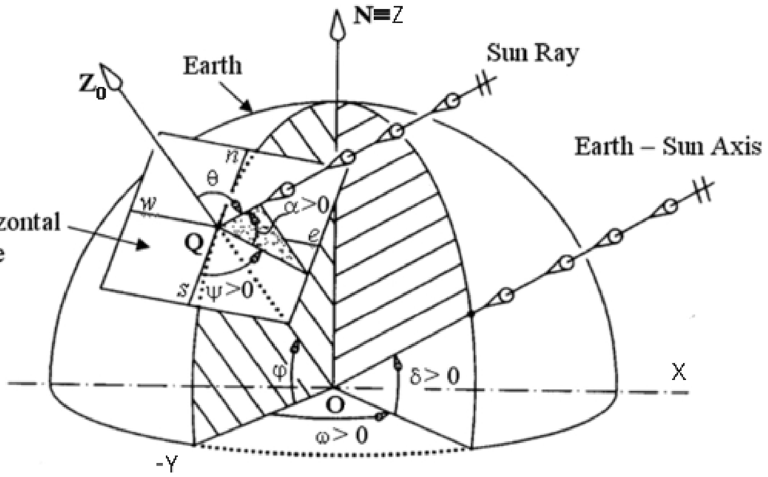

Figure 2. Reference systems (OXYZ and $QX_0Y_0Z_0$ with $X_0 = e$ and $Y_0 = n$) and solar angles (latitude φ, declination δ, hour angle ω, altitude α, azimuth ψ, zenith θ) used in the sunray modelling.

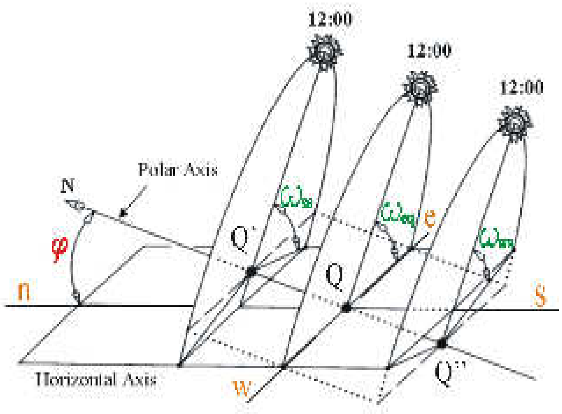

Figure 3. Variation of the sunrise and sunset hour angle

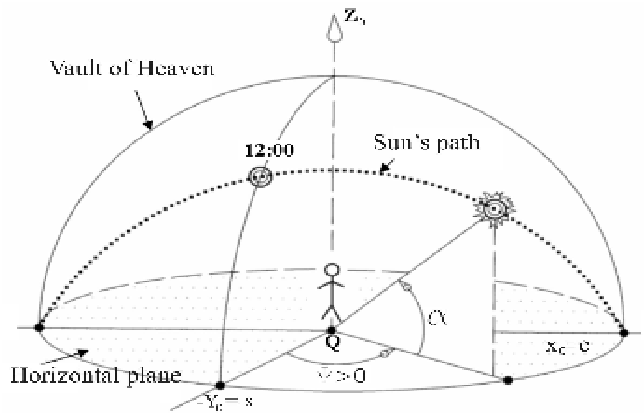

Figure 4. Altitude and Azimuth Angle

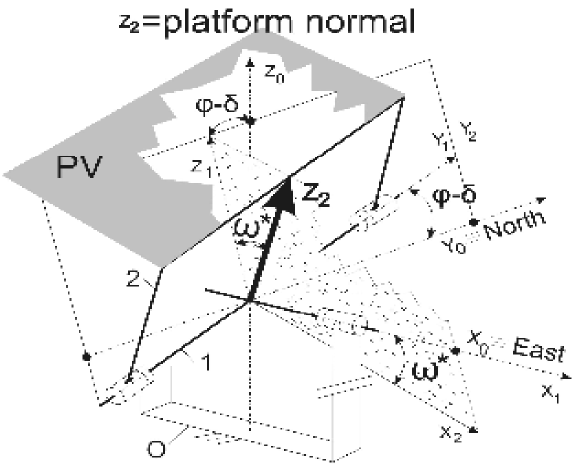

Figure 5. Kinematical scheme of the (pseudo) equatorial dual-axis tracker

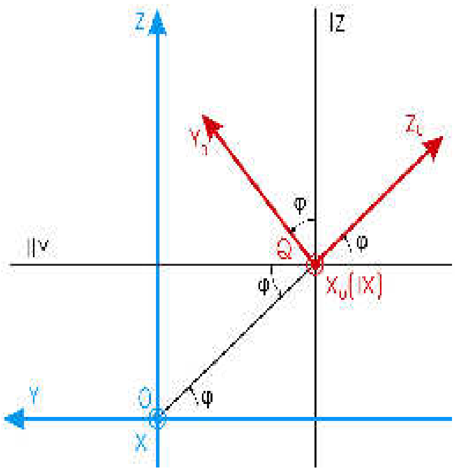

Figure 6. The relative position of the earth reference system Oxyz and the observer reference system $Ox_0y_0z_0$

In figures 1.a and 1.b two representative schemes of how the equatorial dual-axis tracker functions are presented. Figure 1.a presents a view from the global system also depicting a part of the planet Earth, whereas figure 1.b shows the tracker (with its actuators A_Φ and A_ω) as it is seen from the local system.

By means of figures 2, 5, 6, the unit vectors for the normal to the solar panel and the sunray can be calculated:

$$[\bar{e}_{sr}]_{xyz} = \begin{bmatrix} \cos\delta\cdot\sin\omega \\ -\cos\delta\cdot\cos\omega \\ \sin\delta \end{bmatrix}; \qquad (1)$$

$$[\bar{e}_{sr}]_{x_o y_o z_o} = \begin{bmatrix} \cos\alpha\cdot\sin\psi \\ -\cos\alpha\cdot\cos\psi \\ \sin\alpha \end{bmatrix}. \qquad (2)$$

Relation (1) represents the sunray unit vector in the equatorial system OXYZ, whereas (2) represents the same unit vector in the azimuthal system $QX_0Y_0Z_0$.

Their comparison is possible when the both unit vectors in the same system are depicted. Therefore expression (1) is transformed in the $QX_0Y_0Z_0$ system by means of figures 2 and 6. This transformation is modelled in the relation (3):

$$[\bar{e}_{sr}]_{x_0 y_0 z_0} = T\cdot[\bar{e}_{sr}]_{xyz} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\varphi & \cos\varphi \\ 0 & -\cos\varphi & \sin\varphi \end{bmatrix} \cdot \begin{bmatrix} \cos\delta\cdot\sin\omega \\ -\cos\delta\cdot\cos\omega \\ \sin\delta \end{bmatrix} = \qquad (3)$$

$$= \begin{bmatrix} \cos\delta\cdot\sin\omega \\ -\cos\delta\cdot\cos\omega\cdot\sin\varphi+\sin\delta\cdot\cos\varphi \\ \cos\delta\cdot\cos\omega\cdot\cos\varphi+\sin\delta\cdot\sin\varphi \end{bmatrix}_{x_o y_o z_o}$$

By equalizing relation (1) and (3), both representing the sunray unit vector in the $QX_0Y_0Z_0$ system, the next three correlations are obtained:

$$\cos\alpha\cdot\sin\psi = \cos\delta\cdot\sin\omega; \qquad (4)$$

$$-\cos\alpha\cdot\cos\psi = -\cos\delta\cdot\cos\omega\cdot\sin\varphi+\sin\delta\cdot\cos\varphi; \qquad (5)$$

$$\sin\alpha = \cos\delta\cdot\cos\omega\cdot\cos\varphi+\sin\delta\cdot\sin\varphi. \qquad (6)$$

From relation (5) the expression of the altitude (α) can be very simple obtained:

$$\alpha = \sin^{-1}(\sin\delta\cdot\sin\varphi+\cos\delta\cdot\cos\varphi\cdot\cos\omega). \qquad (7)$$

The expression of the azimuth angle ψ can be obtained using the relations (4) and (5). These calculations lead to the conclusion that the *azimuth angle* has the following expression:

$$\psi = (\operatorname{sgn}\omega)\cos^{-1}\frac{\sin\alpha\cdot\sin\varphi-\sin\delta}{\cos\alpha\cdot\cos\varphi}, \qquad (8)$$

where (sgnω) is used for having a positive azimuth until noon and negative from noon until evening.

For the equatorial system the expressions for the used angles are taken from the literature [1, 2]:

Declination:

$$\delta = 23.45^o \sin\frac{360^o(n-80)}{365}; \qquad (9)$$

Hour Angle:

$$\omega = 15^o(12-T). \qquad (10)$$

Formula (9) and (10) are used in the equatorial system, whereas (7) and (8) are used in the azimuthal system. Another important formula is the one which indicates the *hour angle* at which the sun rises and sets (see figure 3), depending on the season and the latitude [1]:

Sunrise, Sunset Hour Angle:

$$\omega_{sr,ss} = \pm\cos^{-1}(-\tan\varphi\tan\delta). \qquad (11)$$

The normal unit vector to the panel is depicted in relation (12) and is calculated with the help of figures 2 and 5.

$$[\bar{e}_{PV-eq}]_{x_0 y_0 z_0} = \begin{bmatrix} \sin\omega^* \\ -\sin(\varphi-\delta)\cdot\cos\omega^* \\ \cos(\varphi-\delta)\cdot\cos\omega^* \end{bmatrix}. \qquad (12)$$

The figure 7,a,b,c,d exemplifies, by numerical simulations, the variations of the previous angles for the latitude of 45°N, taking into consideration the three most important days of a year: summer solstice (top curves), equinoxes (middle curves) and winter solstice (bottom curves). Figure 3 helps to understand the variations from the figure 7, a, b, c, d.

3. Incidence Angle

The *incidence angle* Sun-PV panel, depicted as the angle between the sunray and the normal to the PV panel, results as the "dot product" of previous two unit vectors (the sunray unit vector and the PV panel normal unit vector depicted in the correlations (3) and (12)):

$$\cos\nu = \bar{e}_{sr}\cdot\bar{e}_{PV-eq} = \cos\alpha\cdot\sin\psi\cdot\sin\omega^* + \\ +\cos\alpha\cdot\cos\psi\cdot\cos\omega^*\cdot\sin(\varphi-\delta)+ \\ +\sin\alpha\cdot\cos\omega^*\cdot\cos(\varphi-\delta) \qquad (13)$$

For economical reasons, the hour angle of the panel ω* is modified (for instance) only once every hour, while the other angle (φ-δ) remains constant.

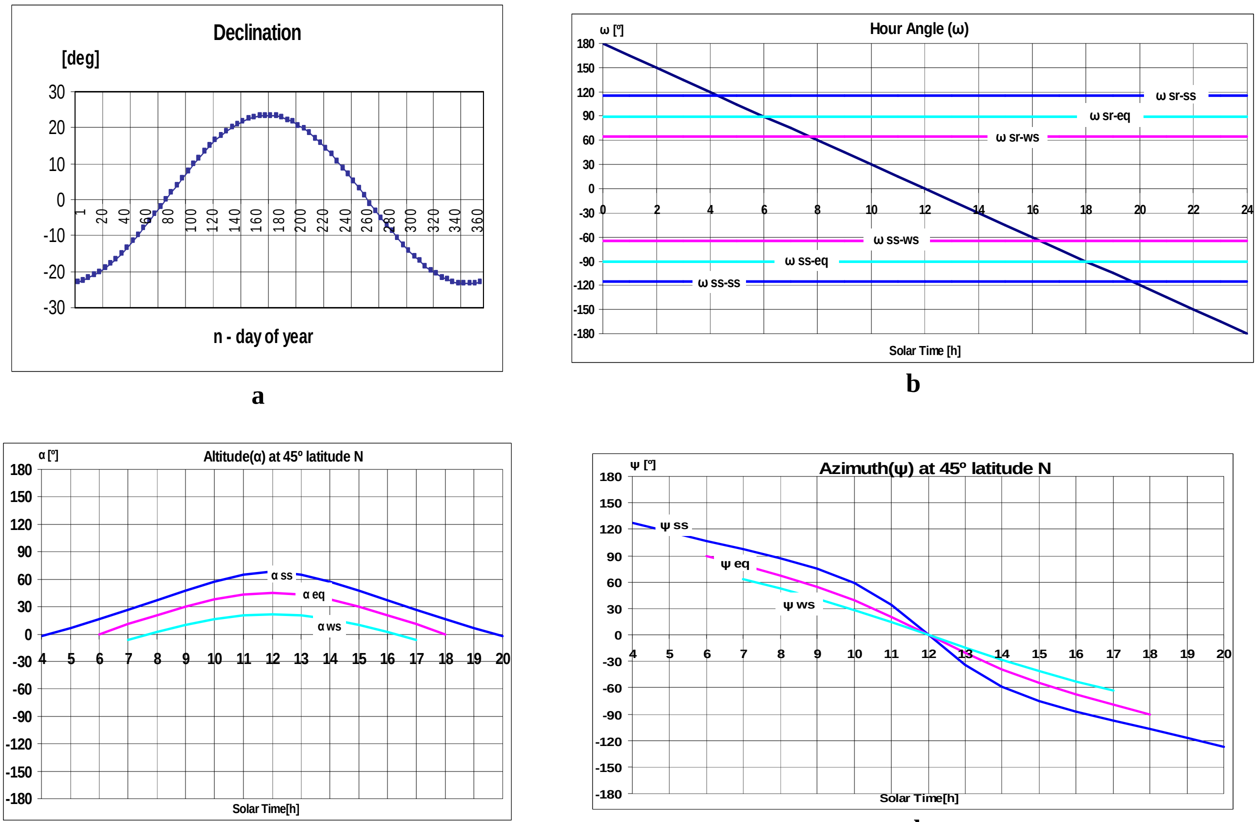

Figure 7. Variations of the declination during a year (a) and variations of the solar angles: hour angle, altitude and azimuth during the summer solstice, equinoxes and winter solstice (b, c, d)

In the figures 8, 9 and 10 there are exemplified by numerical simulations the variations of the auxiliary angles (see figures 8, 10 and 12) and of the incidence angle for the tracked panel, horizontal fixed and tilted fixed panels, at 45°N latitude during the summer solstice, equinoxes and winter solstice (see figures 9, 11, 13).

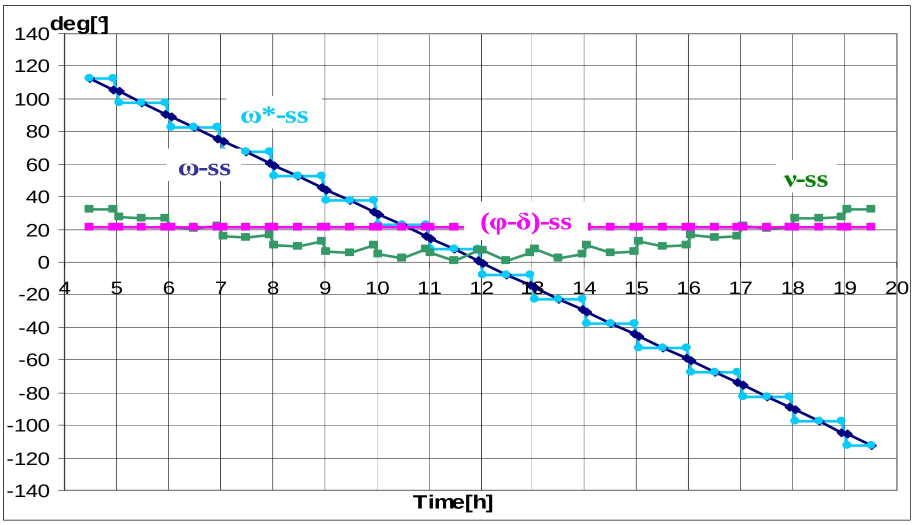

Figure 8. Variations of the (pseudo)equatorial tracker angles during the summer solstice

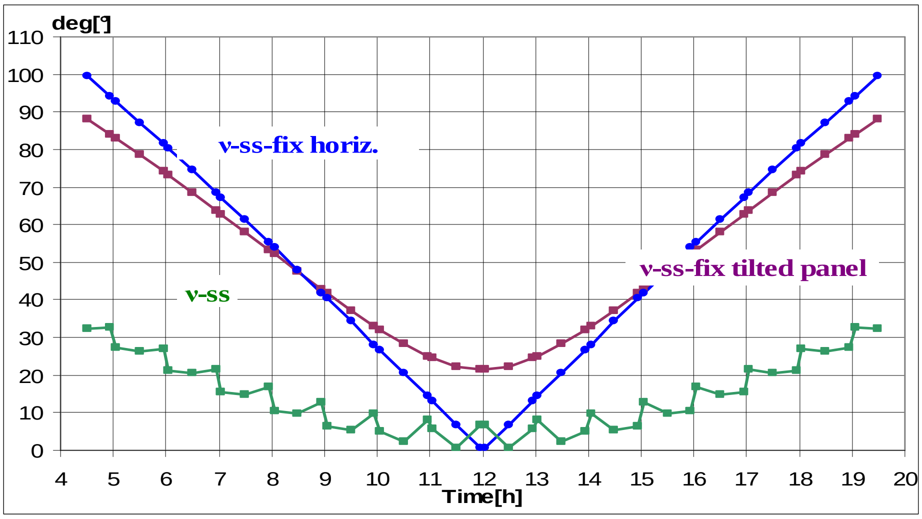

Figure 9. Hourly variation of the incidence angle for three cases: fixed horizontal panel, fixed tilted panel and tracked panel during the summer solstice

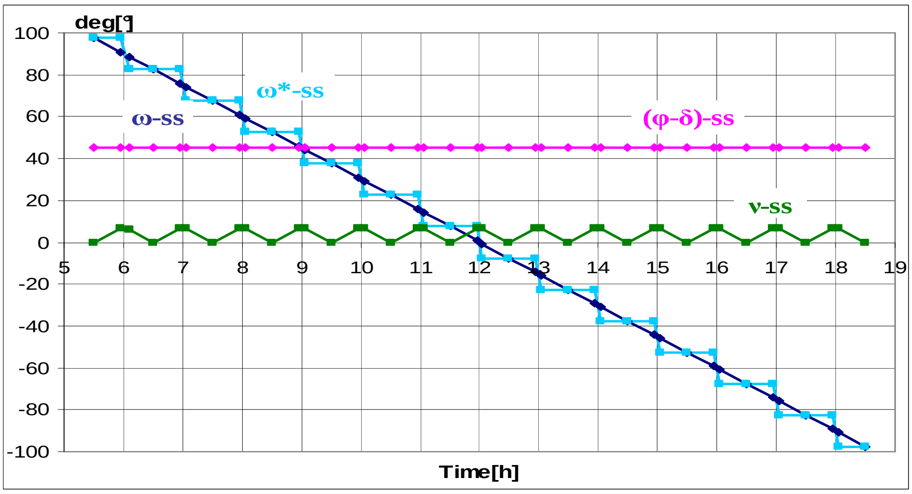

Figure 10. Variations of the (pseudo)equatorial tracker angles during the equinoxes

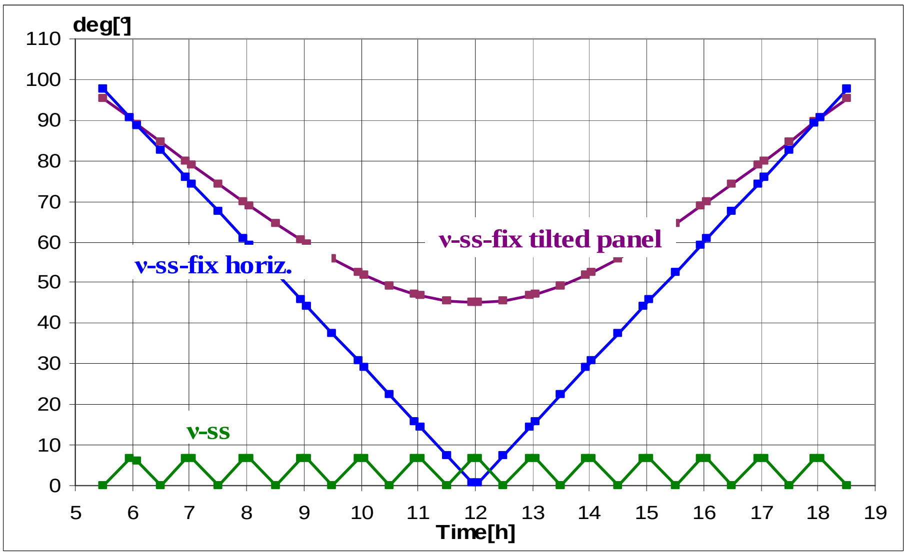

Figure 11. Hourly variation of the incidence angle for three cases: fixed horizontal panel, fixed tilted panel and tracked panel during the equinoxes

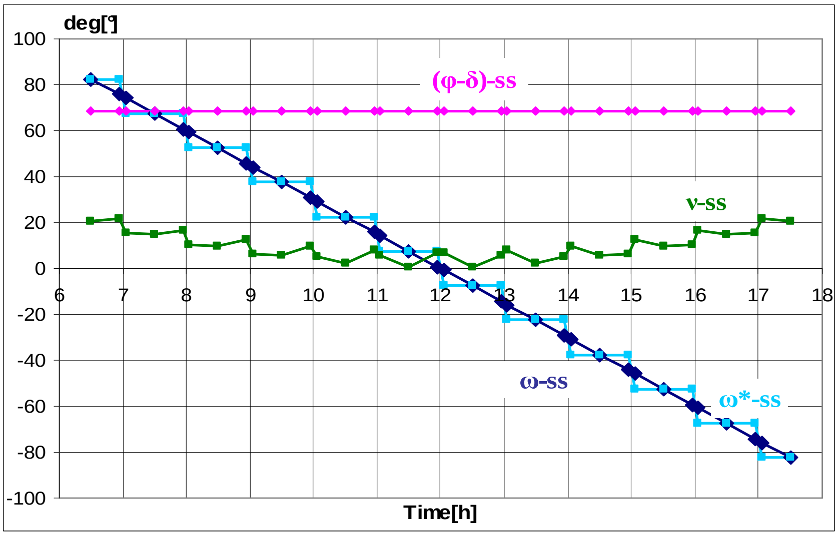

Figure 12. Variations of the (pseudo)equatorial tracker angles during the winter solstice

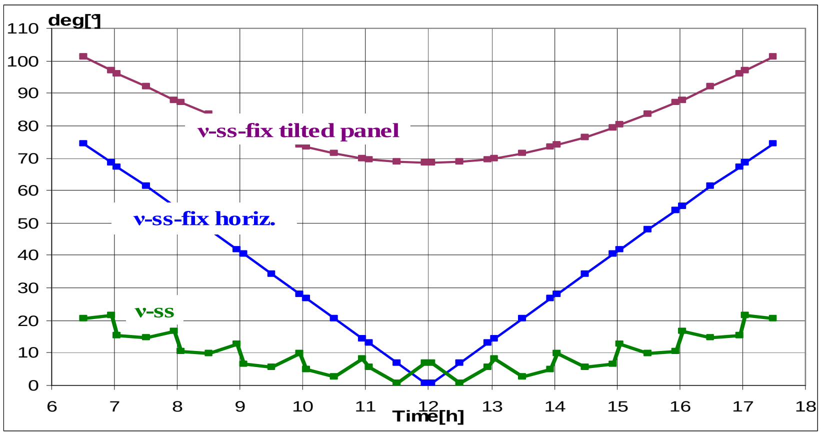

Figure 13. Hourly variation of the incidence angle for three cases: fixed horizontal panel, fixed tilted panel and tracked panel during the winter solstice

4. Conclusions

In this first part of the paper, the incidence angle model of the (pseudo)equatorial PV tracker was elaborated; this model has been numerically simulated with the help of the Excel software.

From the comparative analysis of the incidence angle of the tracked PV panel vs. the fixed PV panels, it becomes obvious that the incidence angle for the (pseudo)equatorial tracker is far lower; this means that the PV panel orientation is very efficient and advisable in practice.

Despite of the fact that the simulations of the (pseudo)equatorial tracker show that during the morning and evening the incidence angle is bigger, its influence is quite little over the received solar radiation (this aspect will be analyzed in one of the next papers).

In the second part of the paper, the azimuthal PV tracker will be analyzed and a comparison between these two tracker types will be made.

References

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