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KINEMATICAL AND DYNAMIC PROPERTIES OF A SPEED MULTIPLIER USED IN WIND TURBINES

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Abstract. Representative kinematical and dynamic aspects of a planetary speed multiplier for wind turbines are presented in the paper: analysis of the transmission ratio and efficiency and analysis of the dynamic response in a relevant functioning case.

Keywords: speed multiplier, efficiency, multiplying ratio, wind turbine, kinematical analysis, dynamic response

1. Introduction

Most wind turbines drive trains (figure 1) include a gearbox to increase the speed of the input shaft to the generator. An increase in speed is needed because the wind turbines rotors turn at a much lower speed than is required by most electrical generators. The range in which the input angular speed must be increased is 5 … 30 [1]. There are two basic types of gearboxes used in wind turbines applications: parallel-shaft gearboxes and *planetary gearboxes*. In order to obtain higher values of the transmission ratio, multiple stages are placed in series. This arrangement applied for the first type of gearbox increases the transmission ratio but, in the same time, increases the overall dimension. The *planetary gearboxes* have a number of significant differences from the first type: the input and output shafts are coaxial, that reduces the radial and axial dimensions; there are multiple power branches, so the loads on each gear are reduced; the gearboxes are relatively light and compact.

Some gearboxes have a secondary function such as supporting the main shaft bearings. Therefore, these gearboxes are designed partially or fully integrated, the main shaft and the main shaft bearings being integrated in the rest of the gearbox.

Taking into account these considerations, the paper main objective is to analyze a partially integrated planetary speed multiplier (figure 2), which is used in wind turbines' applications, from kinematical and dynamic point of view.

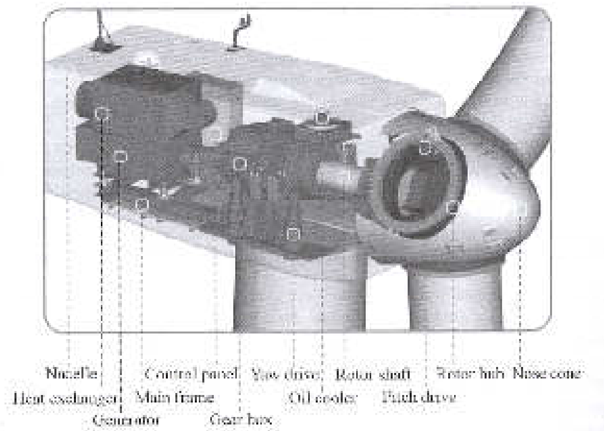


Figure 1. The main components of a wind turbine

2. Structural and Kinematical Aspects

A two-stage planetary multiplier used in wind turbines is represented in figure 2a while in figure 2b it is represented its structural scheme. The low-speed shaft *h1* (the main shaft), supported by bearings, is rigidly connected to a planet carrier, which holds three identical small gears (satellites) mounted on short shafts and bearings. These gears mesh to a large ring gear and a small sun gear, forming a planetary unit that works as a speed multiplier. In order to increase the kinematical multiplying ratio, a second planetary unit of the same type, is serially connected to the first one: the sun gear *1* drives the high speed carrier *h2*, to which it is by a teeth coupling connected. The high speed shaft *5* is supported by bearings mounted in the case (figure 2 a and b). The turbine rotor is attached to the low-speed shaft *h1* while the generator is coupled to the high-speed shaft and is also bolted to the case.

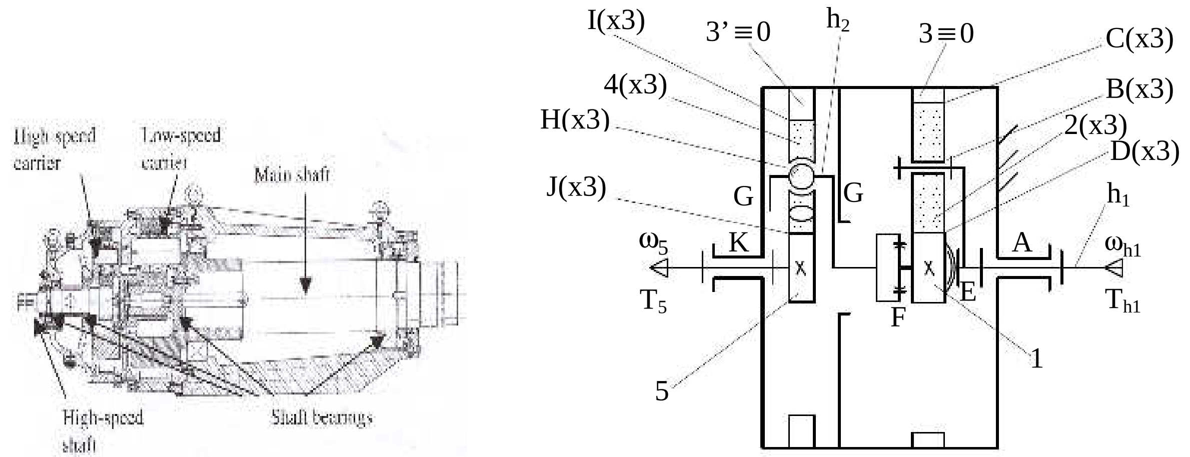


Figure 2. A planetary speed multiplier used in a wind turbine: embodiment scheme (a) and structural scheme (b)

The gearbox has two external links: an input *h1* from the turbine and an output *5* to the generator. The mechanism's degree-of-freedom is given by the following relation (see figure 2b):

$$M = 6n - [\sum c_i - c^*] = \\ = 6 \cdot 10 - [(5 \cdot 5 + 5 \cdot 3 + 6 \cdot 2 + 7 \cdot 1) - 0] = 1, \quad (1)$$

in which n = 6 is the number of elements, c_i – joint *"i"* degree of constraint (there are 5 joins with c_i = 5: A, B(x3) and K; 5 joints with c_i = 3: F, G and H(x3); 6 joints with c_i = 2: C(x3) and D(x3); 7 joints with c_i = 1: E and H(x3)) and c^*= 0 - the number of redundant constraints. Thus, the gearbox is characterized through:

M = 1 → an external independent motion:
(φ_{h1}, $\omega_{h1} = d\varphi_{h1}/dt$, $\varepsilon_{h1} = d\omega_{h1}/dt$);
→ a transmission function for forces:
$T_{h1} = T_{h1}(\varphi_{h1}, T_5)$.

L–M = 1 → a transmission function for motions:
$\varphi_5 = \varphi_5(\varphi_{h1})$, $\omega_5 = d\varphi_5/dt$, $\varepsilon_5 = d\omega_5/dt$;
→ an external independent force: T_5.

The gearbox transmission function for motions can be obtained by using the planetary gearbox kinematical ratios. Therefore, according to figure 2b and under the prerequisite that the associated fixed axes units (obtained by reversing the motion relative to the carriers *h1* and *h2*, respectively) have the *internal kinematical ratios*:

$$i^I_0 = i^{h1}_{1,3} = \omega_{1,h1}/\omega_{3,h1} = -z_3/z_1 = -3{,}5, \\ i^{II}_0 = i^{h2}_{5,3'} = \omega_{5,h2}/\omega_{3',h2} = -z_{3'}/z_5 = -4 \quad (2)$$

for the analyzed speed multiplier the following *kinematical transmission ratio* is obtained:

$$i_{h1-5} = \frac{\omega_{h1,3}}{\omega_{5,3'}} = i_I .. i_{II} = \frac{\omega_{h1,3}}{\omega_{1,3}} \cdot \frac{\omega_{h2,3'}}{\omega_{5,3'}} =$$

$$i^3_{h1-1} . i^{3'}_{h2-5} = 1/[(1 - i^{II}_0)(1 - i^I_0)] = \\ = 1/[5 \cdot 4.5] = +1/22.5$$

$$\omega_5 = \omega_{5,3'} = \omega_{h1,3}/i = \omega_{h1}/i = 22.5 \cdot \omega_{h1} \quad (3)$$

in which i_I and i_{II} are the kinematical ratios of the two planetary units.

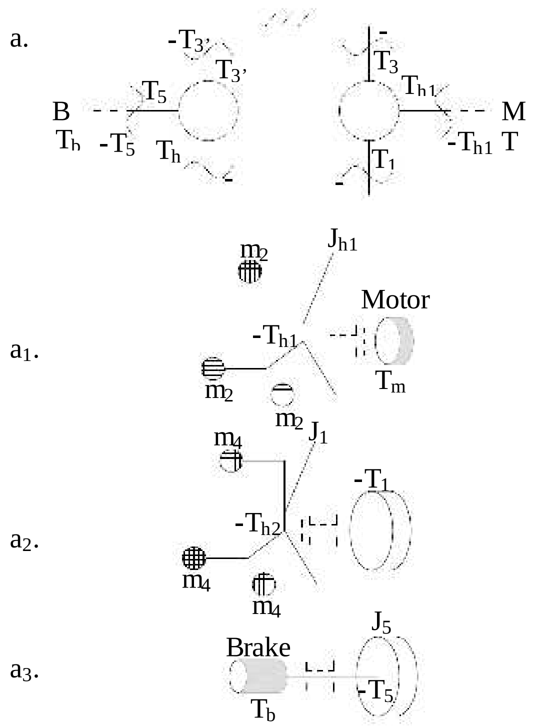


Figure 3. Schemes used in the dynamic analysis by Newton-Euler method

Taking into account relation (3), the analyzed planetary gearbox *multiplies the input speed ω_{h1} 22.5 times* and offers it as output speed ω_5; therefore, the gearbox is obviously a torque reducer.

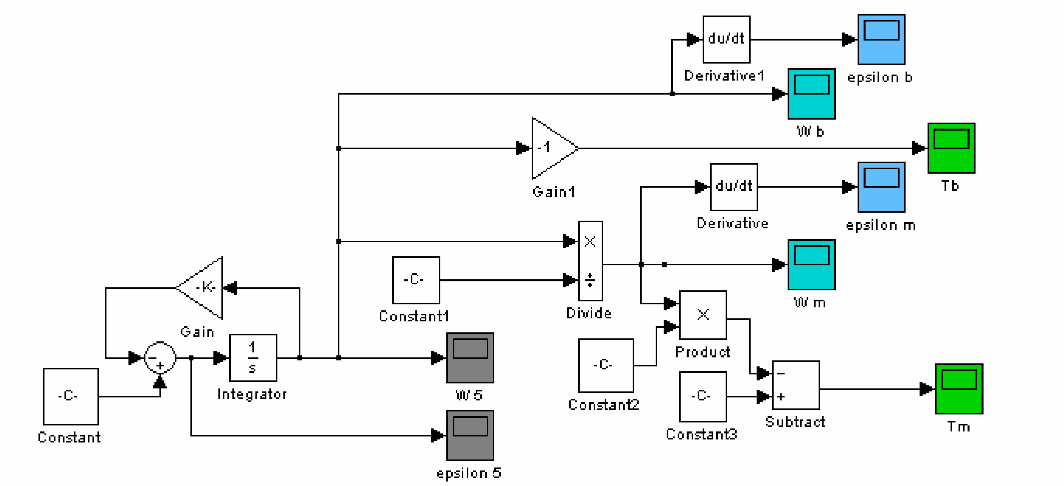
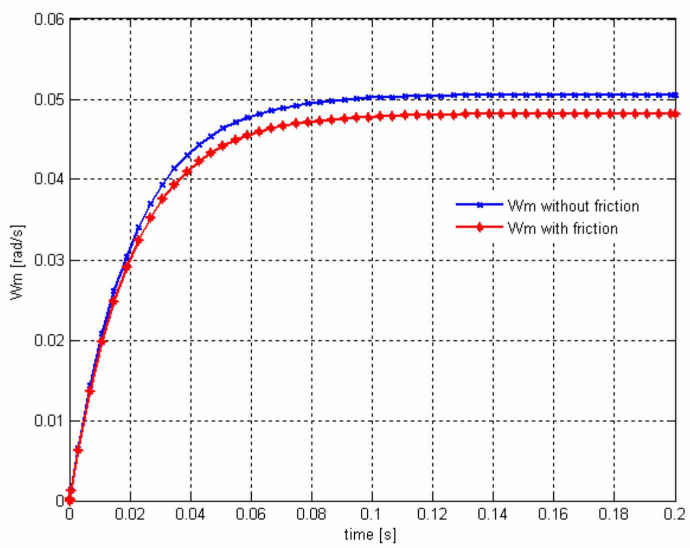


Figure 4. The Simulink scheme which models the motion equation of the planetary speed multiplier

a.



a_1.

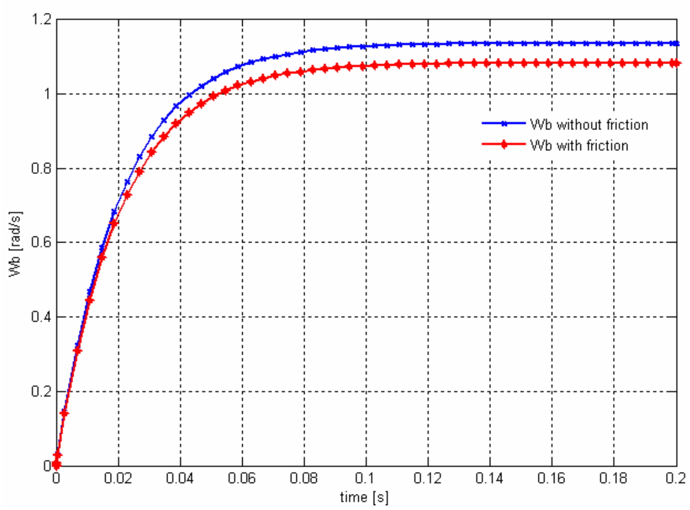
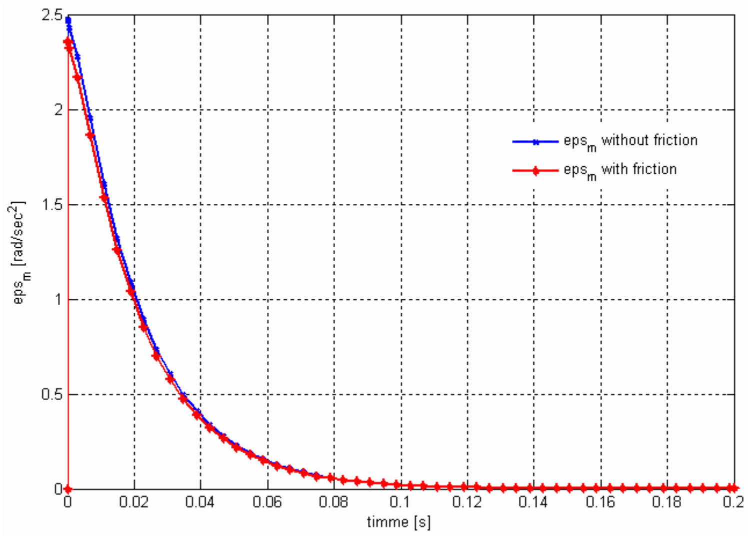
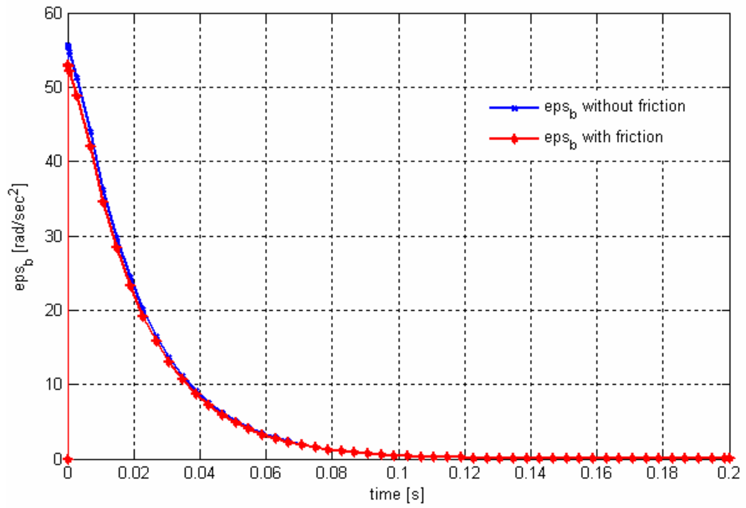


Figure 5 a, a_1. Dynamic response of the aggregate (DC motor+multiplier+brake): input angular velocity (a), output angular velocity (a_1)



b.



b_1.

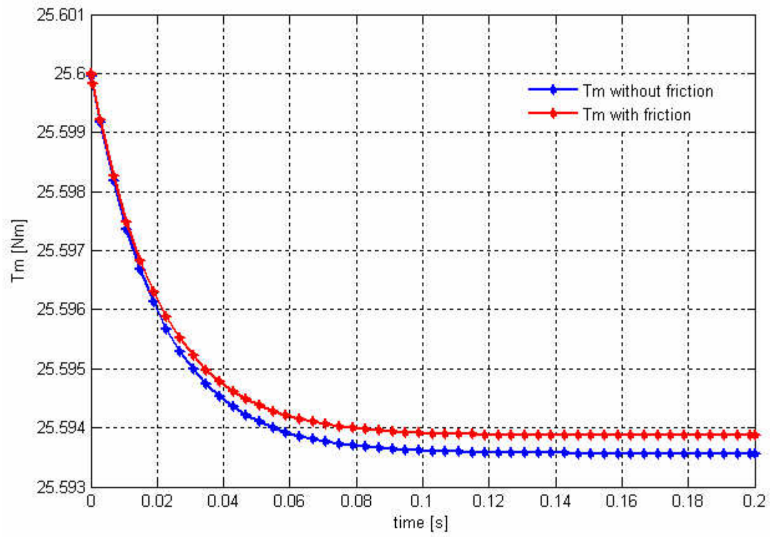
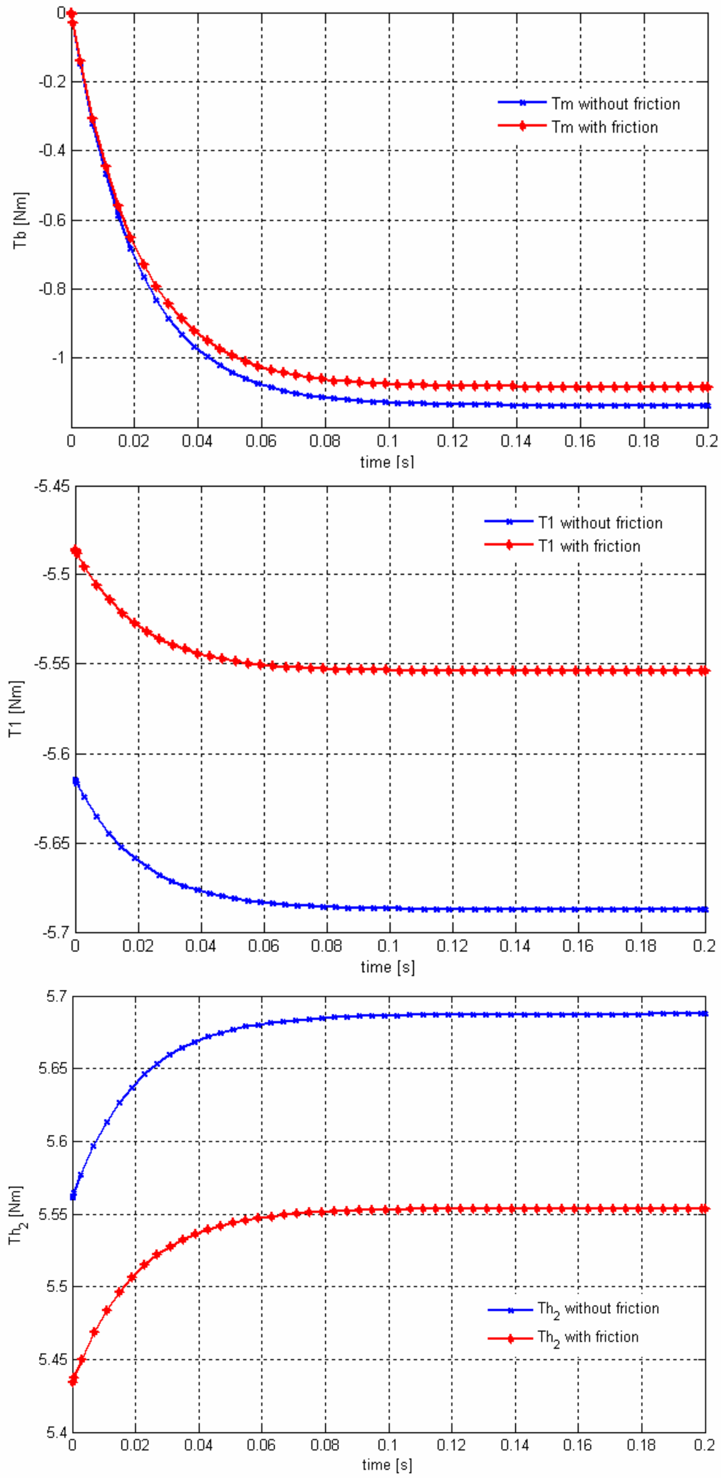


Figure 5 b, b_1, c. Dynamic response of the aggregate (DC motor + multiplier + brake): input angular acceleration (b), output angular acceleration (b_1), input torque (c)



d.

e.

f.

Figure 5,d, e, f. Dynamic response of the aggregate (DC motor+multiplier+brake): output torque (d) and torques' variations of the body 1 ≡ *h*2 (e) and (f)

3. Dynamic Aspects

Under the prerequisite that the fixed axes units associated to the two planetary gears are characterized by the internal efficiencies [2, 3]:

$$\eta_{0I} = \eta^{h1}_{1,3} = \eta^{h1}_{1,2}\eta^{h1}_{2,3} = 0.985^2 = 0.97,$$
$$\eta_{0II} = \eta^{h2}_{5,3'} = \eta^{h2}_{5,4}\eta^{h2}_{4,3'} = 0.985^2 = 0.97 \qquad (4)$$

and the following power flow exponents [1;3]:

$$w^I = \mathrm{sgn}(\omega_{1,h1}T_1) = \mathrm{sgn}[(\omega_{1,h1}T_1)/(-\omega_{1,3}T_1)]$$
$$= \mathrm{sgn}[-i^I_0/(i^I_0-1)] = \mathrm{sgn}[+3,5/(-3,5-1)] = -1$$
$$w^{II} = \mathrm{sgn}(\omega_{5,h2}T_5) = \mathrm{sgn}[(\omega_{5,h2}T_5)/(-\omega_{5,3'}T_5)]$$
$$= \mathrm{sgn}[-i^{II}_0/(i^{II}_0-1)] = \mathrm{sgn}[+4/(-4-1)] = -1$$

Consequently, the gearbox has the efficiency:

$$\eta = \frac{-\omega_{5,3'}T_5}{\omega_{h1,3}T_{h1}} = \frac{-T_5}{T_{h1}}\frac{1}{i} = \frac{-T_5}{T_{h1}}[(1-i_0^{II})(1-i_0^{I})]$$

$$= \frac{(1-i_0^{II})(1-i_0^{I})}{(1-i_0^{II}\eta_{0II}^{wII})(1-i_0^{I}\eta_{0I}^{wI})} = \frac{22.5}{24.00358} = 0.9529 \qquad (5)$$

$$\Rightarrow \quad T_5 = -T_{h1}\eta . i = -\frac{T_{h1}}{22{,}5}0.9529 = -T_{h1}/23.61133$$

That means that the planetary gearbox *reduces the input moment 23.61133 times*.

For the dynamic modelling (see figure 3), an aggregate composed from a DC motor, the analysed planetary speed multiplier and a brake is considered (that means that the analysed planetary speed multiplier is mounted into a experimental testing stand); this modelling relies on the Newton-Euler's method and on the following premises:

a) the elements are rigid bodies and are made of steel;
b) the rotational inertial effects from the satellites (considered lonely - figure 2a) are neglected;
c) the rubbing effect is considered by means of the efficiency η;
d) the inertia moments are considered vs. the axes that passes their masspoints (figure 3): the aggregate rotor+shaft *h1*+satellites *2*: J_{h1}=0.03 kgm^2; the aggregate gear *1*+shaft *h2*+satellites *4*: J_1=0.01 kgm^2; the aggregate gear *5*+brake rotor: J_5=0.02 kgm^2;
e) the DC motor and the brake have known mechanical characteristics: the motor is characterized by equation T_m = - 0.1273 · ω_m + 25.6 [*Nm*];
the brake is characterized by a linear mechanical characteristic: $T_b = -\omega_b$ [*Nm*].

The dynamic modelling is made under the following cases:

I. The motion equation is modelled by neglecting rubbing, and
II. The motion equation is modelled by considering rubbing effects.

Thus, the following correlations can be written, taking into account relations 1÷6) and figure 3:

$$\begin{aligned}
&\omega_5 = i\cdot\omega_{h1} = 22.5\cdot\omega_{h1} \Rightarrow \varepsilon_5 = i\cdot\varepsilon_{h1} = 22.5\cdot\varepsilon_{h1}\\
&\omega_1 = \omega_{h2} = i_{1,h1}^3\cdot\omega_{h1} = 4.5\cdot\omega_{h1} \Rightarrow\\
&\varepsilon_1 = \varepsilon_{h2} = i_{1,h1}^3\cdot\varepsilon_{h1} = 4.5\cdot\varepsilon_{h1}\\
&T_1 + T_{h1} + T_3 = 0; \qquad T_5 + T_{h2} + T_{3'} = 0;\\
&T_1\cdot i_0^I\cdot\eta_{0I}^{w1} + T_3 = 0; \;\; T_5\cdot i_0^{II}\cdot\eta_{0II}^{w2} + T_{3'} = 0;\\
&J_{h1}\varepsilon_{h1} = T_m - T_{h1}; \; J_1\varepsilon_1 = -T_1 - T_{h2};\\
&J_5\varepsilon_5 = T_b - T_5.
\end{aligned} \qquad (6)$$

After replacements, it outcomes the motion equation:

- for case I (η =1):

$$\varepsilon_5 + 48.8899\cdot\omega_5 - 55.6118 = 0 . \qquad (7)$$

- for case II (η < 1):

$$\varepsilon_5 + 48.9194\cdot\omega_5 - 53.0269 = 0 \qquad (8)$$

By making ε_5 = 0 in relation 7 and 8, it is obtained the angular speed ω_5 in steady-state regime for both cases (neglecting and considering rubbing effects):

- for case I (η =1): $\omega_5 = +1.1375\ s^{-1}$
- for case II (η <1): $\omega_5 = +1.0839\ s^{-1}$.

Solving equations (5) and (6) with *Matlab-Simulink* software (see figure 4), the variation in time of the angular speed ω_5, the angular acceleration ε_5 and the input and output torques (see figure 5) are obtained for both cases.

4. Conclusions

1. The analyzed planetary gearbox increases the input speed 22.5 *times* and decreases the input moment 23.61133 *times* (see figure 5).
2. The dynamic modeling is made in the premise that the speed multiplier is used in a system of type: motor+multiplier+brake (see figure 3).
3. In the case of neglecting the rubbing effects, the output angular speeds ω_5 are bigger than in the case of considering friction (but with insignificant values, see figure 5 a), while the external torques' behaviour (Tm and Tb, see figure 5 c, d) is reversed.
4. The variations of the angular acceleration ε_5, with and without rubbing, are practically superposed (figure 5b).
5. The system consisting of motor, speed multiplier, brake starts practically, in both cases, in about 0.1 s, after which enters in the steady-state regime.
6. In the steady-state regime, the torques which load the body 1 ≡ *h*2 (see figure 3 a2) become equal as modulus (see figure 5 e and f).

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