# MATHEMATICAL METHODS TO DETERMINATE THE UNFOLDING OF THE CYLINDERS

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**Abstract.** This paper establishes the intersection curves between cylinders, by using the Mathematic 5.1. program. This thing can be obtained by introducing the curves equations, which are inferred, in Mathematic 5.1 program. This paper takes into discussion two cylinders.

**Keywords:** unfolding, Mathematic 5.1, intersection curve, cylinder, projection vertical

## 1. Introduction

The calculation of the unfolding has a wide applicability, especially in the connection of the pipes with equal of different diameters. This calculation can be done by using several methods among which: the use of classical methods, the use of analytical methods, the use of mathematical methods with a view to automating their drawing and numerical methods using the "spline". For illustration, we took two cylinders with the following diameters: *C* of diameter *D* = 48 mm and $C_1$ of diameter $D_1$ = 30 mm. We know the angle $\varphi = 45^o$, too.

## 2. The mathematical method of establishing the intersections curves

The projection of the intersection curves necessitates the solving of the following phases:
- the writing of the curves equations resulted from the intersections of the areas that can be unfolded;
- the writing of the transformations equations by the unfolding of the intersection curve.

### 2.1. The calculation of the intersection curve $\gamma_1$ of the cylinder C and $\gamma_2$ of the cylinder $C_1$.

In accordance with figure 1 we take the cylinder *C*, of diameter *D*, and its reference system Oxyz and the cylinder $C_1$, of diameter $D_1$,and its reference system $O_1x_1y_1z_1$, where $y \equiv y_1$ and $O \equiv O_1$.

The cylinders equations expressed in the chosen reference systems are:

$$x^2 + y^2 = R^2 \quad (1)$$

$$y_1^2 + z_1^2 = R_1^2 \quad (2)$$

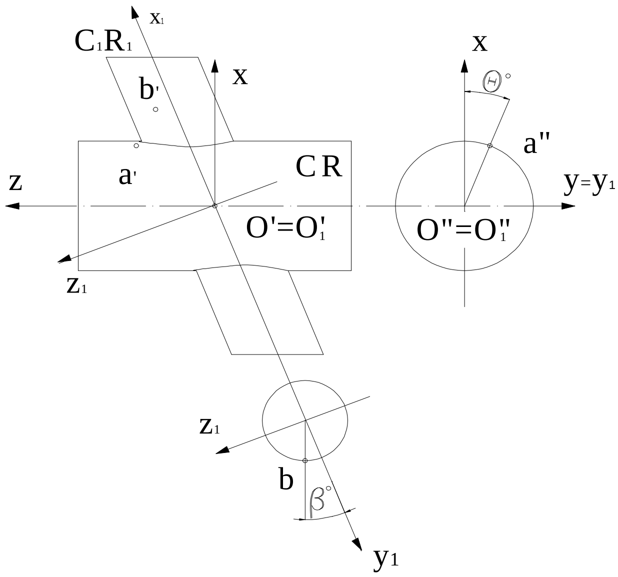

Figure 1. The geometrical elements of the cylinders

The two reference system are rotated, one given another, by the angle$\varphi$. The transformation formula of the coordinates, to passing from the system Oxyz into $O_1x_1y_1z_1$ and viceversa are:

$$x_1 = x\cos\varphi + z\sin\varphi \quad (3)$$

$$z_1 = z\cos\varphi - x\sin\varphi \quad (4)$$

$$x = x_1\cos\varphi - z_1\sin\varphi \quad (5)$$

$$z = x_1\sin\varphi + z_1\cos\varphi \quad (6)$$

We relate the equations of the both cylinders to system Oxyz and by eliminating the variable y, we obtain the equation of the vertical projection of the intersection:

$$z^2 - 2x \cdot tg\varphi \cdot z + \frac{R^2 - R_1^2}{\cos^2\varphi} - x^2 = 0 \quad (7)$$

The equation of the transformation curve $\gamma_1$, border of the cylinder *C*, is obtained by applying the transformations (8, 9) to the equation (7).

$$x = R\cos\theta = R\cos\frac{x_d}{R} \quad (8)$$

$$z = z_d \quad (9)$$

where $x_d$ and $z_d$ are the coordinates of the point A in unfolding. This point A is indicated by its projections a' and a".

In this case the following equation is obtained:

$$z_d^2 - 2Rz_d\cos\frac{x_d}{R}\cdot tg\varphi + \left[\frac{R^2 - R_1^2}{\cos^2\varphi} - R^2\cos^2\frac{x_d}{R}\right] = 0 \quad (10)$$

Then:

*Plot[{24*Cos[x/24]*Tan[45 Degree] + Sqrt[225-576*(sin[x/24])^2]/Cos[45 Degree], 24*Cos[x/24] *Tan[45Degree] - Sqrt[225- 576* (sin[x/24])^2]/ Cos[45 Degree]}, {x,-24* Arcsin[15/24],24* Arcsin[15/24]}]*

$$z_{d1,2} = R\cos\frac{x_d}{R}tg\varphi \pm \frac{1}{\cos\varphi}\sqrt{R_1^2 - R^2\sin^2\frac{x_d}{R}} \quad (11)$$

$$x_d \in \left[-R\cdot\arcsin\frac{R_1}{R}, R\cdot\arcsin\frac{R_1}{R}\right]$$

We obtain the figure 2, by introducing the relations (11) into Mathematic 5.1 program.

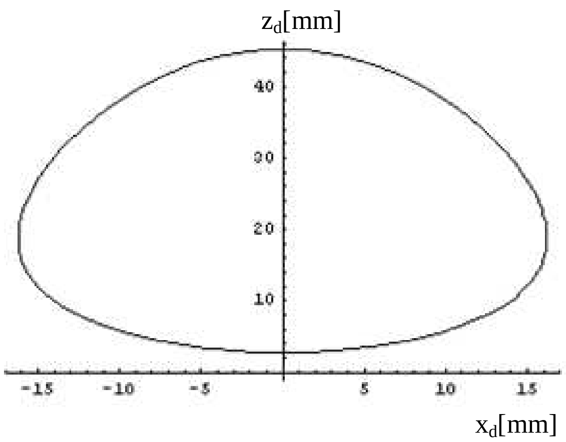


Figure 2. The unfolding of the intersection curve $\gamma_1$ of the cylinder *C*

The equation of the transformation curve $\gamma_2$, border of the cylinder $C_1$, is obtained by applying the transformations (12, 13) to the equation (7):

$$x_1 = x_{d1} \quad (12)$$

$$z_1 = R_1\sin\alpha = R_1\sin\frac{z_{d1}}{R} \quad (13)$$

where $x_{d1}$ and $z_{d1}$ are the coordinates of the point B(b', b) in unfolding.

The following equation is obtained:

$$x_{d1}^2 + 2R_1\sin\frac{z_{d1}}{R_1}x_{d1} - R_1^2\sin^2\frac{z_{d1}}{R_1} - \frac{R^2 - R_1^2}{\cos^2\varphi} = 0 \quad (14)$$

Then:

$$x_{d1} = -R_1\sin\frac{z_{d1}}{R_1} \pm \frac{1}{\cos\varphi}\sqrt{R^2 - R_1^2\cos^2\frac{z_{d1}}{R_1}} \quad (15)$$

$$z_{d1} \in [0, 2\pi R_1] \quad (16)$$

The figure 3 is obtained by introducing the relations (15, 16) into Mathematic 5.1 program.

*Plot[{-15*sin[z/15] + Sqrt[576-225*(cos[z/15])^2/ Cos[45 Degree],-15*Sin[z/15]-Sqrt[576-225* (Cos[z/15])^2]/Cos[45Degree]}, {z,0,2*PI*15}]*

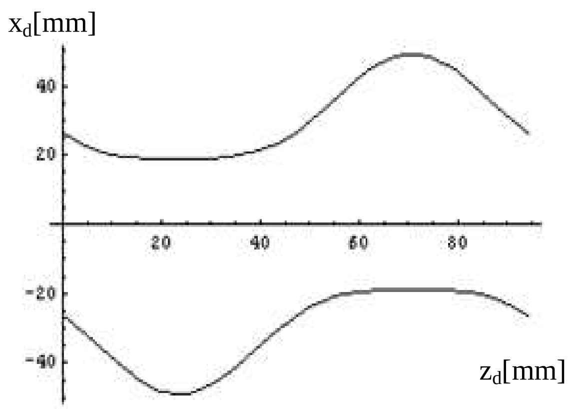


Figure 3. The unfolding of the intersection curve $\gamma_2$ of the cylinder $C_1$

**2.2. The calculation of the intersection curve $\gamma_1$ of the cylinder *C* and $\gamma_2$ of the cylinder $C_1$, using another mathematical method.**

The situation vector of any point, which is placed on the cylinder *C* (figure 4), is:

$$\vec{\rho} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k} \quad (17)$$

where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors of the axes of the choice reference system.

But *x, y, z* are the coordinates of a point which is situated on the cylinder:

$$x = R\sin\theta$$
$$y = R\cos\theta \qquad (18)$$
$$z = z$$

where θ is the angle of the vectorial radius with Oy.

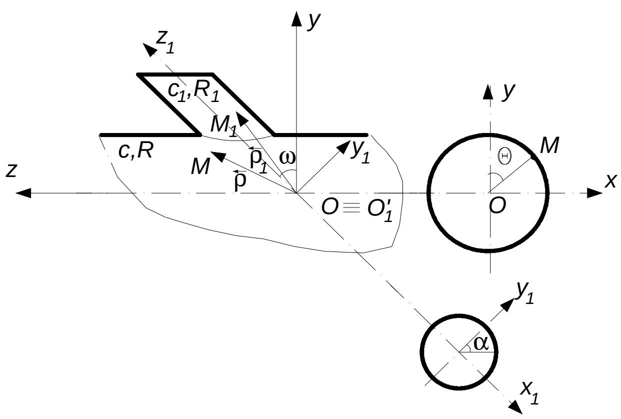


Figure 4. The geometrical elements of the cylinders

Then:

$$\vec{\rho} = R\sin\theta\,\vec{i} + R\cos\theta\,\vec{j} + z\,\vec{k} \qquad (19)$$

The situation vector of a point, which is placed on the cylinder $C_1$, is:

$$\vec{\rho}_1 = x_1\,\vec{i} + y_1\,\vec{j}_1 + z_1\,\vec{k}_1 \qquad (20)$$

where $\vec{i}_1, \vec{j}_1, \vec{k}_1$ are the unit vectors of the rotated system $O_1x_1y_1z_1$. The coordinates $x_1y_1z_1$ of a point placed on the cylinder $C_1$ are:

$$x_1 = R_1\sin\alpha$$
$$y_1 = R_1\cos\alpha \qquad (21)$$
$$z_1 = z_1$$

where α is the angle between the vectorial radius with $O_1x_1$. The unit vectors of the coordinates axes $O_1x_1y_1z_1$ can be expressed depending on the unit vectors of the system Oxyz thus:

$$\vec{i}_1 = \vec{i}$$
$$\vec{j}_1 = \vec{j}\sin\omega - \vec{k}\cos\omega \qquad (22)$$
$$\vec{k}_1 = \vec{k}\sin\omega + \vec{j}\cos\omega$$

Then, the situation vector of any point placed on the cylinder $C_1$, expressed given the reference system Oxyz is:

$$\vec{\rho}_1 = R_1\cos\alpha\,\vec{i} +$$
$$+ (z_1\cos\omega + R_1\sin\alpha\sin\omega)\,\vec{j} + \qquad (23)$$
$$+ (z_1\sin\omega - R_1\sin\alpha\cos\omega)\,\vec{k}$$

For all the intersection points of the cylinders $C$ and $C_1$, the vectors $\vec{\rho} = \vec{\rho}_1$, therefore their projections on the axes of the reference system are likewise:

$$R\sin\theta = R_1\cos\alpha$$
$$R\cos\theta = z_1\cos\omega + R_1\sin\alpha\sin\omega \qquad (24)$$
$$z = z_1\sin\omega - R_1\sin\alpha\cos\omega$$

From (24) we obtain:

$$z_1 = R\frac{\cos\theta}{\cos\omega} - R_1\sin\alpha\cdot tg\omega \qquad (25)$$

Therefore:

$$z_1 = \pm\frac{\sqrt{R^2 - R_1^2\cos^2\alpha}}{\cos\omega} - R_1\sin\alpha\, tg\omega \qquad (26)$$

$$\alpha \in [0, 2\pi] \qquad (27)$$

Is obtained the same figure, like in the previous case, by introducing the relations (26, 27) into Mathematic 5.1 program.

## References


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