MONOWHEEL DYNAMICS

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Abstract. The present paper deals with the dynamics problem of a vehicle having a single wheel – the monowheel. Because of the particular kind of contact link between the vehicle and road, the dynamics of such a vehicle is not very simple. The authors present the way to find the motion equations, which are commented, in order to offer a better understanding of such a vehicle motion.

Keywords: monowheel, motion, equations

1. Introduction

The monowheel history begins in the second half of the 19th century, when there were manufactured some vehicles, driven by pedals. The first motor-driven monowheel was the "petrol monocycle" of Caravaglia presented in 1904 at Milan Exposition (figure 1). The principle is easy to understand from the figure. The big wheel with rim and tyre contains a frame which may rotate relatively to the rim, being fitted with ball-bearings at its corners.

Figure 1. Caravaglia's monowheel

The frame supports both the driver, and the petrol engine: the rim is toothed on the side, and engages with the pinion of the engine. In sum, the mobile wheel rolls around the built fixed engine, and of course, around the driver!

Another example is the modern monowheel built by Dr.Geraint Owen of Bath University, U.K. (figure 2).This monowheel consists of a 7ft metal hoop, with a 50 cc moped engine, driver's seat and controls mounted in an inner frame [1].

Figure 2. Owen's monowheel

We might note the existence even of a monowheel driven by a Buick V8 engine: this is a vehicle built by Kerry Maclean in the 2000's.

2. Vehicle motion

2.1. Simplified model

From the figures 2 and 3, we can see that the vehicle is a multi-body system composed by:

- the inner body 1, composed, at its turn, by a frame ABDC, holding three small wheels 4, the driving wheel 3, and of course the engine and, finally, the driver;

- the outer wheel 2, composed by a rim with its tyre [2].

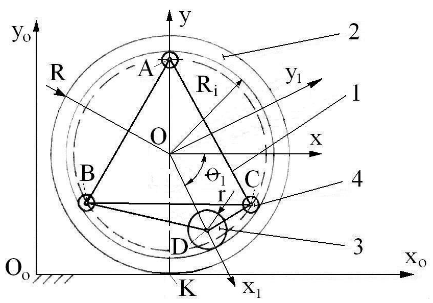

Figure 3. Simplified 2D-model

2.2. Reference frames

For the simplest motion, meaning the straight forward motion, we only need the following three reference systems (Fig.3):

- a fixed reference frame $O_0x_0y_0$;
- a transported reference frame *Oxy*, contained in the symmetry plane of the outer wheel 2, but having the *Ox*-axis always horizontal;
- a mobile reference frame Ox_1y_1, bound to the inner body 1, and having the Ox_1-axis passing by the point D.

2.3. External forces and couples

To begin with, we shall make some simplifying assumptions. First, we shall consider the speed small enough in order to be able to neglect the air resistance. The second assumption is that the centre of gravity G of the system has an unchanged position relatively to the frame Ox_1y_1, meaning that its coordinates α and $\rho = OG$ are constant.

But, that does not mean a constant position of this point in the transported frame *Oxy*, because there is a relative rotation of the inner body by the angle θ_1 (Fig.4).

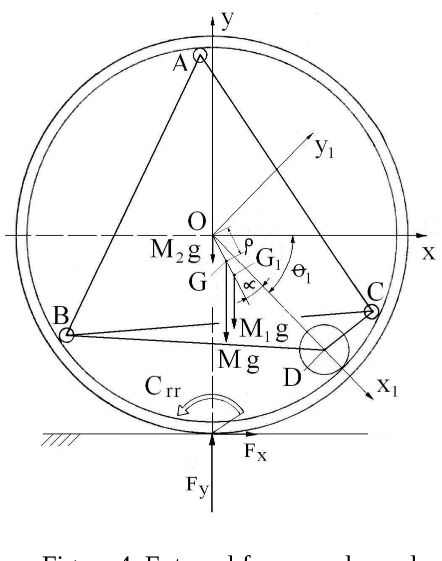

Figure 4. External forces and couples

We shall consider the following notations:

- M_1, J_1, the mass of the inner body 1, and its moment of inertia about an axis normal to the plane xOy and passing by the point G_1,
- M_2, J_2, the mass of the outer wheel 2, and its moment of inertia about an axis normal to the plane *xOy* and passing by the point O,
- J_3, the moment of inertia of the driving wheel about its axis of rotation,
- J_e, the moment of inertia of the engine.

Finally, we shall consider only the following external forces:

- the weights Mg, M_1g, M_2g, having evident significations,
- the contact reactions F_x, F_y, meaning tangential force and normal force in the contact point K,

and couples:

- the rolling resistance couple C_{rr}, acting in the same contact point K,
- the driving couple C_e, applied by the engine (Fig.5),

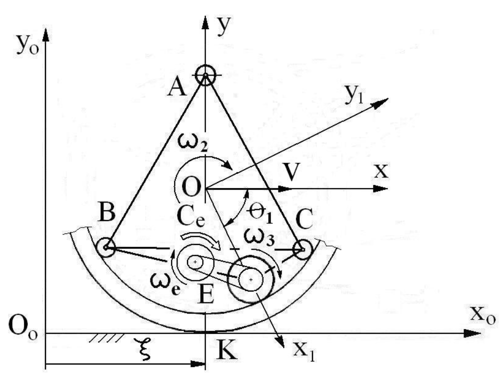

Figure 5. Driving gear

2.4. Kinematic relationships

In order to write the expressions of kinetic energy of the system, and of the mechanical work developed by the acting external forces, we need to establish the kinematic relationships. So, we can write, on the base of figures above:

$$\xi = R\theta_2 , \tag{1}$$

$$x_G = \xi + \rho\cos(\alpha + \theta_1), \tag{2}$$

$$y_G = R - \rho\sin(\alpha + \theta_1). \tag{3}$$

Further, assuming that the movement transmission between the driving wheel 3 and the outer wheel 2 is without slipping, (being realised by a gear with toothed wheels, or by friction wheels without relative slipping), we can write

$$R_i\theta_2 = r\theta_3 , \tag{4}$$

$$\theta_e = i_{e,3}\,\theta_3 . \tag{5}$$

where $i_{e,3}$ represents the gear ratio between engine and driving wheel 3.

These five relations contain seven unknowns: $x_G, y_G, \xi, \theta_1, \theta_2, \theta_3, \theta_e$. That means, as we already pointed out, that the system has two degrees of freedom. Taking, for example, as independent variables θ_1 and θ_2, the other five coordinates will result easily from the equations presented above.

In the particular case of the steady-state rectilinear motion, the position angle θ_1 remains constant in value. Consequently, the y_G-coordinate will be also constant, and so we shall have only four equations between five coordinates $x_G, \xi, \theta_2, \theta_3, \theta_e$. In that case, the system will have only one degree-of-freedom (as in the case of an ordinary motorcycle). Such a system is a so-called *desmodromic system.*

2.5. Kinetic energy

The kinetic energy of the entire vehicle is the sum between the kinetic energies of its constituents:

$$T = \frac{1}{2}\left\{ \begin{array}{l} Mv_G^2 + J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2 + \\ + J_e\omega_e^2 \end{array} \right\}, \tag{6}$$

where:

$$v_G^2 = \dot{x}_G^2 + \dot{y}_G^2 , \tag{7}$$

$$\omega_1 = \dot{\theta}_1 , \tag{8}$$

$$\omega_2 = \frac{\dot{\xi}}{R} , \tag{9}$$

$$\omega_3 = \frac{R_i}{r}\omega_2 = \frac{R_i\dot{\xi}}{rR} , \tag{10}$$

$$\omega_e = i_{e,3}\,\omega_3 = i_{e,3}\frac{R_i\dot{\xi}}{rR} . \tag{11}$$

In the equation (7), we shall introduce the corresponding derivatives as follows

$$\dot{x}_G = \dot{\xi} - \dot{\theta}_1\,\rho\sin(\alpha + \theta_1), \tag{12}$$

$$\dot{y}_G = -\dot{\theta}_1\,\rho\cos(\alpha + \theta_1). \tag{13}$$

Finally, the kinetic energy has the expression

$$T = \frac{1}{2}\left\{ \begin{array}{l} M\left[\begin{array}{l} R^2\dot{\theta}_2^2 + \dot{\theta}_1^2\rho^2 - \\ - 2\dot{\theta}_1 R\dot{\theta}_2\,\rho\sin(\alpha + \theta_1) \end{array} \right] + \\ + J_1\dot{\theta}_1^2 + J_2\dot{\theta}_2^2 + \\ + J_3\left(\frac{R_i}{r}\right)^2\dot{\theta}_2^2 + \\ + J_e\, i_{e,3}^2\left(\frac{R_i}{r}\right)^2\dot{\theta}_2^2 \end{array} \right\} \tag{14}$$

From the above expression, it is now evident that the independent coordinates are θ_1 *and* θ_2.

2.6. Mechanical work

The mechanical work done by external forces will be:

$$W = C_e\theta_e - C_{rr}\theta_2 - Mg\,\rho\left[1 - \sin(\alpha + \theta_1)\right]. \tag{15}$$

where, in accordance with (1) to (5), we shall put:

$$\theta_e = i_{e,3}\frac{R_i}{r}\theta_2 . \tag{16}$$

Then, in terms of independent coordinates, the mechanical work will be

$$\begin{array}{l} W = C_e\, i_{e,3}\frac{R_i}{r}\theta_2 - C_{rr}\theta_2 - \\ - M\,g\,\rho\left[1 - \sin(\alpha + \theta_1)\right]. \end{array} \tag{17}$$

Here, we have to note that *the external forces F_x and F_y will not develop mechanical work*, because they are always acting in a point at instantaneous rest (the contact point K) [3]. Evidently, this fact stands on the assumption that there is no sliding of the wheel on the road.

3. Differential equations of motion

3.1. Lagrange's equations

The Lagrange equations have the form [4]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = \frac{\partial W}{\partial q_j}, \quad j = 1,2 \tag{18}$$

Here, the generalized coordinates are

$$q_1 = \theta_1 , \; q_2 = \theta_2 . \tag{19}$$

Taking the corresponding derivatives, required by equations (18), we get the following two differential equations:

$$\left(M\rho^2 + J_1\right)\ddot{\theta}_1 - 2M\rho R\dot{\theta}_1\dot{\theta}_2 \cos(\alpha+\theta_1) - \\ - M\rho R\ddot{\theta}_2 \sin(\alpha+\theta_1) = Mg\rho\cos(\alpha+\theta_1), \tag{20}$$

$$M\rho R\ddot{\theta}_1 \sin(\alpha+\theta_1) + \ddot{\theta}_2 \begin{bmatrix} MR^2 + J_2 + \\ + J_3\left(\dfrac{R_i}{r}\right)^2 + \\ + J_e\, i_{e,3}^2\left(\dfrac{R_i}{r}\right)^2 \end{bmatrix} = \\ = C_e\, i_{e,3}\left(\frac{R_i}{r}\right) - C_{rr}\,. \tag{21}$$

4. Conclusions

1. Evidently, these two differential equations can be integrated only in a numerical way. However, even without integration, a simple qualitative analysis can give some useful information about the motion.

First, we can easily see that in the simplest case of motion, namely in the steady-state plane motion, these equations give

$$C_e\, i_{e,3}\frac{R_i}{rR} - C_{rr}\frac{1}{R} = 0\,, \tag{22}$$

$$Mg\,\rho\cos(\alpha+\theta_1) = 0\,. \tag{23}$$

From the equation (22), we can find the minimum torque of the engine, necessary to ensure the uniform motion at small speeds:

$$C_e = \frac{C_{rr}\, r}{i_{e,3}\, R_i} = \frac{Mg\, s\, r}{i_{e,3}\, R_i} \tag{24}$$

Further, from the equation (23) we find that, in steady-state conditions at low speed, the centre of mass of the inner body 1 must be placed on the vertical line passing by the centre *O* of the outer wheel 2. Indeed, $\cos(\alpha+\theta_1)=0$ leads to this conclusion.

2. When one is studying an actual situation, the formulation of the problem is more complicated, of course. We must take into account the air resistance, which is a function of the speed having the form

$$F_{air} = k\,S\,v^2\,, \tag{25}$$

where k is an aerodynamic coefficient, S is the frontal area of the assembly vehicle-rider, and v is the speed.

3. Moreover, the study has to be extended to the accelerating or braking conditions. These conditions are more difficult for the driver, because if under accelerating conditions his rotation to the back is more or less unpleasant, under extremely braking conditions, the outer wheel may drive the inner body together with the driver, in a complete overturn. All these extreme conditions must be avoided by design and manageable by the driver.

4. Last but not least, the problem of lateral stability (ensured by gyroscopic effect), gives another field to future studies.

5. References

1. ***: http://www.dself.dsl.pipex.com. Accessed: 2009-02-14
2. Botezatu, D., Deliu, G.: *The Study and the Design of the Monowheel Vehicle Model using CAD software.* Proceedings of COMEC 2007 2nd International Conference, Transilvania University of Brasov, p. 213-216, ISBN 978-973-598-117-4, October 2007, Brasov, Romania
3. Deliu, G.: *Mechanics for Engineering Students.* Albastra Publishing House, ISBN 973-650-082-9, Cluj-Napoca, 2002
4. Dragos, L.: *Principiile mecanicii analitice (Analytical Mechanics Principles).* Tehnica Publishing House, Bucuresti, 1976 (in Romanian)