# IMPROVING THE BEHAVIOR QUALITY OF EXCITABLE MEDIA PHENOMENA USING RECENT COMPUTATIONAL TOOLS

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**Abstract** Nowadays, computational fluid dynamics (CFD) becomes more and more mature. Its software tools are increasing in importance, and constitute a great challenge for the research in this area. A modern area where CFD has large applications is the mixing theory. Since the years ’48, three basic stages were clearly identified in the study of the mixing:

- The *entrainment* stage – where the large-scale flow structure need to be correctly described;
- The subsequent *stirring* process (responsible for the interfacial surface generation between the mixing species) – where there is necessary, a correct defined intermediate range of scales;
- The *dynamic* process – at the smallest scales – this must be captured to describe the *molecular mixing* process itself.

The mixing problems are still far from complete solving. A recent goal is to find a consistent and coherent theory to stand up that *a mixing model in excitable media leads to a far from equilibrium model.* In this order, the present paper continues the recent work in the field of computational dynamics, bringing new useful features based on statistical and computational observations. The appliances of the soft MAPLE11 are handled and their important applications for the study of the models’ behaviour are pointed out.

**Key words:** turbulent mixing, vortic flow, rare event, Maple Assistant

## 1. Recent advances in excitable media. Flow kinematics

The problems of turbulence were recently approached in a special manner, in [1]. It concerns, on one hand, a special vortex technology which offers a lot of applications in all fields of bio-engineering, especially in processing the polluted fluids, and on the other hand, the mathematical and computational models used for handling this phenomena.

Starting from the importance of implementation of some optimized technologies for processing the polluted fluids, the benefits of this technology are both of scientific and technologic type [1, 6]. It concerns, one one-hand, finding new physic- mathematical models for describing at optimal parameters the turbulent mixing created by a vorticity structure, and from technological standpoint, *developing the vortex technology for handling the polluting materials.*

The statistical idea of flow is generally represented by the map:

$$x = \Phi_t(X), X = \Phi_{t=0}(X) \quad (1)$$

In the continuum mechanics, the relation (1) is named *flow*, and it is a diffeomorphism of class $C^k$. Moreover, (1) must satisfy the relation:

$$0 \langle J \langle \infty,\ J = \det\left(\frac{\partial x_i}{\partial X_j}\right)$$
$$J = \det(D(\Phi_t(X))) \quad (2)$$

where D denotes the derivation with respect to the reference configuration, in this case **X**. The relation (2) implies two particles, $X_1$ and $X_2$, which occupy the same position **x** at a moment. Non-topological behaviour (like break up, for example) *is not allowed*.

With respect to **X** there is defined the basic measure of deformation, the *deformation gradient*, **F,** namely:

$$\mathrm{F} = (\nabla_X \Phi_t(\mathrm{X}))^T,\ F_{ij} = \left(\frac{\partial x_i}{\partial X_j}\right) \quad (3)$$

where $\nabla_X$ denotes differentiation with respect to **X**. According to (3), **F** is non singular. The basic measure for the deformation with respect to **x** is the *velocity gradient*.

After defining the basic deformation of a material filament and the corresponding relation for the area of an infinitesimal material surface, we can define the basic deformation measures: the *length deformation* λ and *surface deformation* η, with the relations [4, 5]:

$$\lambda = (\mathrm{C : MM})^{1/2}, \ \eta = (\det F)\cdot\left(\mathrm{C}^{-1} : \mathrm{NN}\right)^{1/2} \quad (4)$$

with **C** (=$\mathbf{F}^T \cdot \mathbf{F}$) the *Cauchy-Green deformation tensor*, and the vectors **M**,**N** are the orientation versors in length and surface respectively. The scalar form for (4), used in practice, is:

$$\lambda^2 = C_{ij} \cdot M_i \cdot N_j, \ \eta^2 = = (\det F)\cdot C_{ij}^{-1} \cdot M_i \cdot N_j \quad (5)$$

with $\sum M_i^2=1$, $\sum N_j^2=1$, the condition for the versors.

The deformation tensor **F** and the associated tensors **C**, $\mathbf{C}^{-1}$ represents the basic quantities in the deformation analysis for the infinitesimal elements.

We say that the flow $\boldsymbol{x}=\Phi_t(\boldsymbol{X})$ has *a good mixing* if the mean values $D(\ln\lambda)/Dt$ and $D(\ln\eta)/Dt$ are not decreasing to zero, for any initial position P and any initial orientations **M** and **N**.

Thus, there is defined the *deformation efficiency in length*, $e_\lambda = e_\lambda\ (\boldsymbol{X},\boldsymbol{M},t)$ of the material element d**X**, as:

$$e_\lambda = \frac{D(\ln\lambda)/Dt}{(\mathrm{D:D})^{1/2}} \le 1 \quad (6)$$

and similarly, the *deformation efficiency in surface*, , $e_\eta = e_\eta\ (\boldsymbol{X},\boldsymbol{N},t)$ of the area element d**A**: in the case of an isochoric flow (the jacobian equal 1), we have:

$$e_\eta = \frac{D(\ln\eta)/Dt}{(\mathrm{D:D})^{1/2}} \le 1 \quad (7)$$

where **D** is the deformation tensor, obtained by decomposing the velocity gradient in its symmetric and non-symmetric part [5].

## 2. The vortex technology for the mixing in excitable media. Recent results

There was developed a special technology, which concerns the investigation of turbulent mixing in a “Tornado” vortex installation, and is able to process the polluted fluids and to provide new useful materials. The installation consists of a vortex tube which is a modified version, at a low pressure (approx. 0.1 bar) of a Ranque-Hilsch tube. The application area is very large, including *collecting, separation and aggregation of the particles* [1, 6]. The spatial and temporal scales proved that the domains could vary, from the laboratory domains to dissipative ones (corresponding to fine structures).

One end of the tube is completely closed and the air is tangentially introduced by the aspiration operating at the other end of the tube. The air enters the installation through the tangential entries and leaves it by the exit to the aspiration source. It is worth noting the air enters the tube as a *swirling flow*. Near the closed end, an *annular vortex structure* generates, where the swirling ratio number (tangential velocity/axial velocity) attains its maximum. We have to mention this *particular swirling flow control* by comparison to the cyclones, centrifuges or other generators of swirling flow.

From physical and technical standpoint, few special mechanisms were performed with this technology [6]. For the present aims it must be noted that special results were obtained by processing the biological fluids, namely using aquatic algae – *Spirulina Platensis*. The basic effect of the vortexation refers to *the fragmentation, at very small spatial scales, of the biological material*. The gradual fragmentation was performed in the vortex tube and an appropriate parameter allows the representation of the *degree of fragmentation* depending on the non-dimensional parameter, for few experimental results.

Recently it has been started a large qualitative analysis [1, 2, 3] of the mixing behaviour. Taking into account the important aim of elaborating a consistent theory in this field, both 2D and 3D (periodic and non-periodic) cases were taken into account. There were tested very few cases. In 3D case, the computational tests matched the experiments [1]. There were used modern appliances of the soft MAPLE11, like the graphic/ plotting builder and solver builder. Comparing the phase-portrait plot builder and the discrete – numeric plot builder offers in fact *an important qualitative analysis of the mixing behaviour in continuous versus discrete time interval.*

In what follows the analysis is focused on 3D case. In [1] there are realized significant cases of the discrete-time computational qualitative analysis. Since in the classic analysis the target is the study of the efficiencies $e_\lambda$ and $e_\eta$ at *successive moments*, the analysis is a discrete one, which aims testing the special events that could appear at various, *random / irrational*, values of the length / surface versors $M_i$, $N_j$. There are represented all the behaviour case of the mixing process, and conclusive remarks are in the section 4.

It must be noticed that in the vortex technology there are tested four classes of phenomena:

- Relative linear phenomena;
- Negative-linear phenomena corresponding to alternate loadings of stretching and folding of the material filaments. From statistic standpoint, these are the most;
- Mixing phenomena, concerning small or large deviations or strong discontinuities. Is the case when suddenly, pieces of filaments are coming off, then the rest of material starts the vortexation from the beginning;
- Rare events, corresponding to the turbulent mixing. Is the case of breaking up of the filaments of the biologic algae.

## 3. Computational analysis of the vortic flow – the mixing standpoint

Let us consider a particular vortical flow induced by three point vortices, on which a small external, time-dependent disturbance is added. In this particular case, the positions of these vortices are stationary in space without convection in the absence of external disturbance. The construction of a vortex triplet is very interesting [7]. The unperturbed stream function in a two-dimensional, inviscid, incompressible flow that is induced by vortices can be expressed as the sum of the stream functions for each point vortex:

$$\Psi = \sum \Psi_j = -\sum \frac{\Gamma_j}{2\pi} \cdot \ln r_j \tag{8}$$

where

$$r_j = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2} \tag{9}$$

is the distance from vortex position $\left(x_j(t), y_j(t)\right)$

The equations of particle motion are given by:

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{\partial \Psi}{\partial y} \\ \dfrac{dy}{dt} = -\dfrac{\partial \Psi}{\partial x} \end{cases} \tag{10}$$

In particular, let us consider here a vertical flow induced by three point vortices:

$$\Gamma_1 = \Gamma_{-1} = \Gamma, \Gamma_0 = -\frac{1}{2} \cdot \Gamma$$

$$\begin{aligned} r_1 &= \sqrt{[x - (x_0 + x_1)]^2 + [y - (y_0 + y_1)]^2} \\ r_{-1} &= \sqrt{[x - (x_0 - x_1)]^2 + [y - (y_0 - y_1)]^2} \\ r_0 &= \sqrt{(x - x_0)^2 + (y - y_0)^2} \end{aligned} \tag{11}$$

The stream function (8) modifies according to the above expressions. Further, if there is assumed an external disturbance in terms of a stream function, $\Psi_p$:

$$\Psi_p = \varepsilon \cdot \frac{1}{2} \cdot \left(x^2 + y^2\right) \cdot \omega \cdot \sin(\omega t), \varepsilon < 1 \tag{12}$$

Thus the resulting stream function for the flow field is the sum of the unperturbed and the perturbed state:

$$\Psi = \Psi_u + \Psi_p \tag{13}$$

Taking into account a non-dimensionalization of all variables [7], the equations (10) read:

$$\begin{cases} \dfrac{dx}{dt} = -\left[\begin{array}{l} \dfrac{2 \cdot (y - y_1)}{(x - x_1)^2 + (y - y_1)^2} + \\ \dfrac{2 \cdot (y + y_1)}{(x + x_1)^2 + (y + y_1)^2} - \dfrac{y}{x^2 + y^2} \end{array}\right] \cdot \\ \varepsilon \cdot y \cdot \omega \cdot \sin(\omega t) \\ \dfrac{dy}{dt} = \left[\begin{array}{l} \dfrac{2 \cdot (x - x_1)}{(x - x_1)^2 + (y - y_1)^2} + \\ \dfrac{2 \cdot (x + x_1)}{(x + x_1)^2 + (y + y_1)^2} - \dfrac{x}{x^2 + y^2} \end{array}\right] - \\ \varepsilon \cdot x \cdot \omega \cdot \sin(\omega t) \end{cases} \tag{14}$$

The above equations determine the motion of fluid particles, in terms of dimensionless parameters ε and ω.

In what follows there is performed a qualitative analysis of the behaviour of the above model. Few cases are taken into account for both ε and ω. For the present aim only a case is exhibited, namely $\omega = 15, \varepsilon = 0.15$. Following the specific MAPLE11 procedures for differential equations [8], there are obtained the plots of the trajectories-solutions, in few simulation cases. The graphics are as follows, the simulation cases are labeled within each figure.

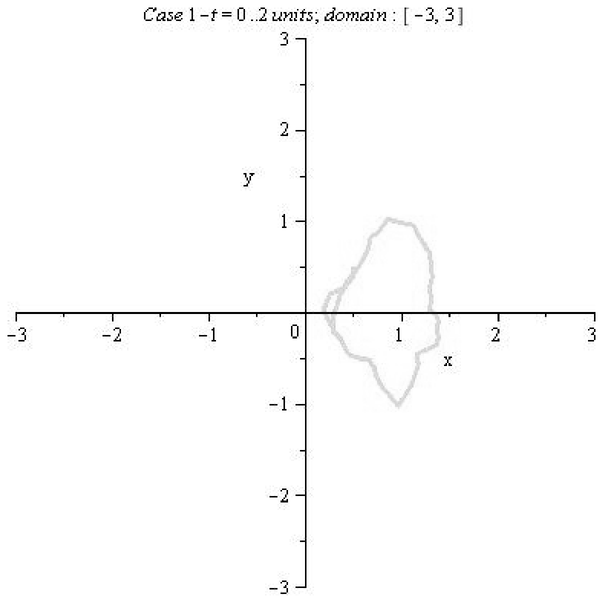

Figure 1.

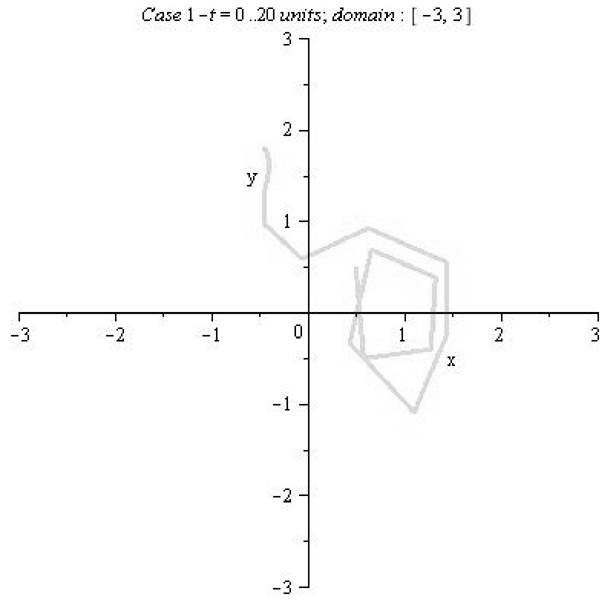

Figure 3.

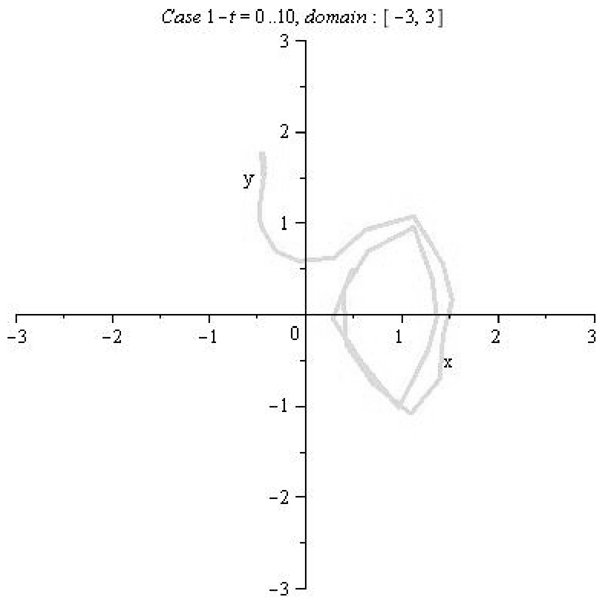

Figure 2.

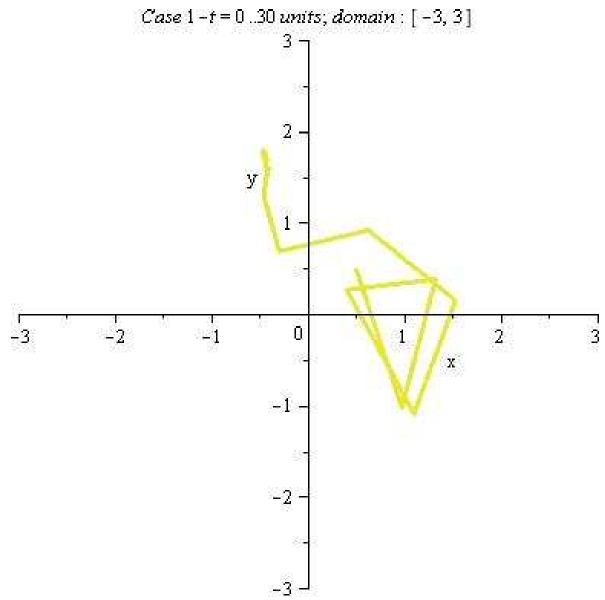

Figure 4.

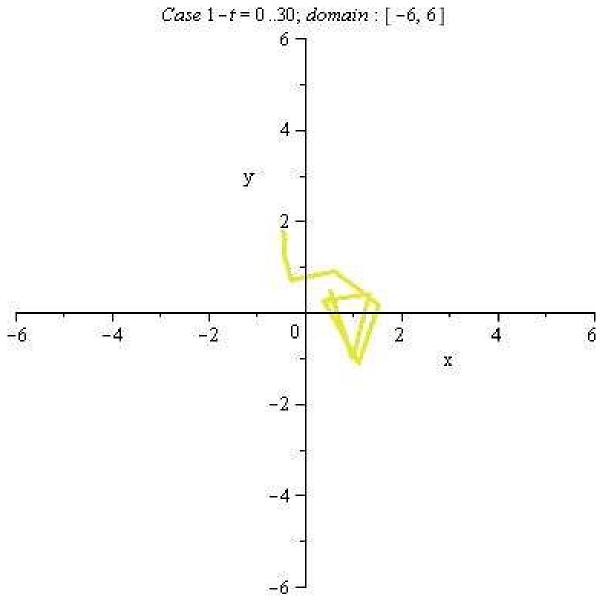


Figure 5.

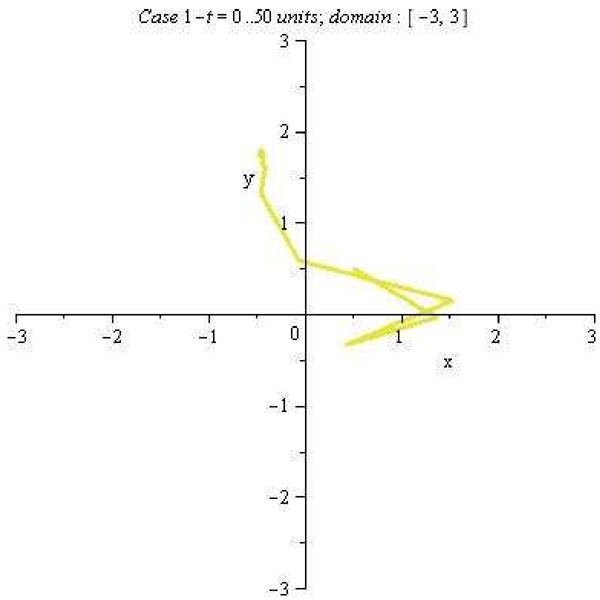


Figure 6.

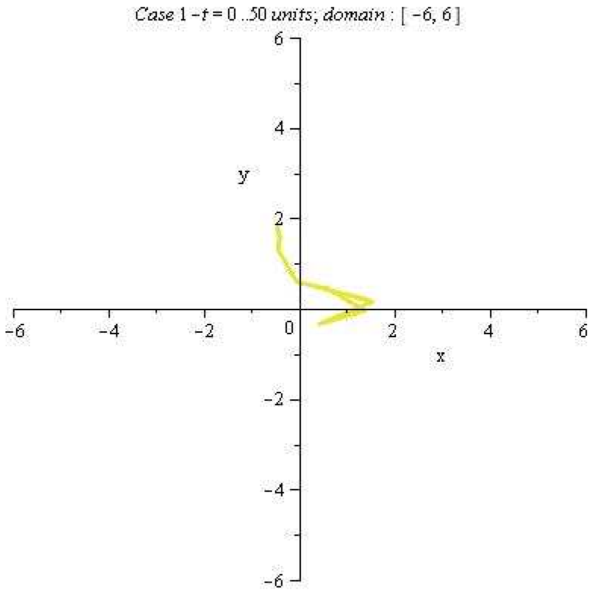


Figure 7.

## 4. Comparing remarks. Conclusions

The remarks are strongly related to the previous work and offer statistical information for the global analysis of the mixing phenomena.

At a first sight, the above graphics seem to look like a limit cycle, but the multiple simulations show that in fact it cannot be assessed that the trajectory behaves like a limit cycle. Although in the model it appears a trend of periodicity – because of the factor $\cos(\varepsilon \cdot (1 - \cos(\omega t)))$ - when increasing the time scale this trend is not regained, in fact there is failed the aspect of limit cycle.

There is verified the fact [7] that the system is symmetric; this is obvious from the graphics. Despite this, when increasing the (symmetric) domain, there is obtained no closed trajectory.

Taking into account only the above two features, it suffices to conclude that the vortic flow proposed is a model *strongly sensitive at initial conditions*; in fact this is a basic feature of *far from equilibrium models*, the models with few freedom degrees; at this point it is regained the global panel of special events – both from modeling and computational standpoint.

If there are analyzed the differences between periodic and a non-periodic flow, the amount of simulations offers interesting remarks. For the non-periodic 3D flow, it must be noticed that a little perturbation has a consistent influence on the model, going into a *far from equilibrium model* [1, 2, 3]. This fact is regained in the above analysis. If there is annexed the irrationality of the length / surface versors values (an appliance used since the beginning of the mixing study), a special panel is obtained, with *random distributed events*. This space-time context consolidates the basic statement that the *turbulent mixing flows must be approached as chaotic systems.*

If there are taken into account the differences between 2D and 3D case, it is easy to deduce the requirement of a special analysis of the influence of parameters on the behavior of this complex mixing flow. This would be achieved by the *spectral analysis*. Together with this aspect it must be also noticed the enlarging of the time scale. In fact, the parameters influence on the model is a quite rich interdisciplinary area, with very few analytic and numeric appliances. It suffices to notice the diverse tools that MAPLE11 offers for the users, and the wide scale of their applications. The vortic flow belongs to this class of requirements, and testing more appliances of MAPLE11 [8] – *such as the interactive plot builder* – offer a rich panel of statistical information.

It is obvious that the testing of more and more values / parameters sets remains basic. If for 3D case, 60 statistical cases seems to suffice for proving the special events favourable for breaking up of the biological filaments, it is expected an acceptance testing of the consistency with the interdisciplinary area. Thus, the issues of *repetitive phenomena*, both in 2D and 3D case, give rises to achieve some appliances of chaotic dynamical systems, whose numeric models would give new research directions on the behaviour in excitable media.

In this order, a next aim is testing more cases for the parameters ε and ω. The vortic flow and the mixing flow are quite complex flows, and therefore in their modeling it must be introduced and used elements of chaotic turbulence in order to obtain and analyze special events.

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