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CONTROL OF A DISCRETE PLANAR TENTACLE ROBOT

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Abstract. The paper deals with a special class of robotic arm, the tentacle robot. First, the discrete dynamic model of the tentacle arm is analyzed. The dynamic model of the hyper-redundant arm with continuum elements in conjunction with the electro-rheological fluid is discussed and the discrete planar model based to the nonlinear observer will be developed. Also, numerical simulations of the arm motion are presented.

Keywords: tentacle robot, discrete control, nonlinear observer

1. Introduction

A tentacle arm is represented by a robot with an infinite number of various flexible elements with distributed parameters (namely the mass and the torque of each element). Many researchers developed important works in this area. Thus, in [1, 2], the mathematical model of tentacle robot is developed, and then the control problem of the movement in spaces with or without restrictions is solved. In [3], the model of such robot with a finite number of elements is presented. Here, an electro-hydraulic controller with more sections is used.

In this paper, a new model of the hyper-redundant robot with a finite number of elements is presented, where each element has an own structure and an independent control circuit, as in figure 1.

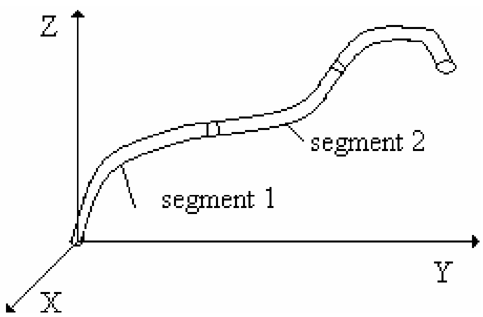
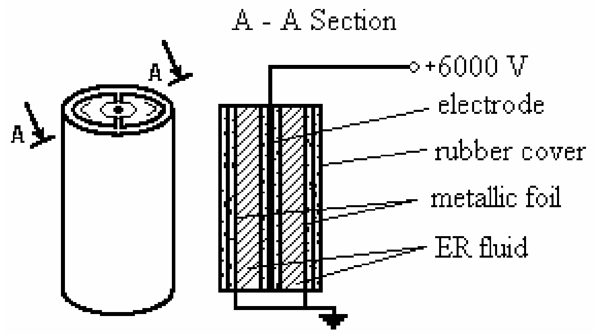


Figure 1. The physical model of a planar tentacle robot

In figure 2, the structure of one tentacle robot segment is shown, that is developed in great detail in [4]. The robot is realized by cylindrical elements which are made by fibbers and reinforced rubber. Inside, each cylinder contains two chambers with electro-rheological fluid (fluids that can dramatically change their properties when a high voltage electrical field is applied) and each chamber has an individual control circuit. Every element of the tentacle structure is controlled simultaneously by two control systems. First system controls the pressure of chambers and the second system controls the viscosity of the electro-rheological fluid. Also, the chambers walls are reinforced with rubber fibbers on radius direction. In this way, the tentacle will be deformed only on axial direction, and the deformation on radial direction being restrained.



a) b)
Figure 2. The physical structure of one element of tentacle robot: (a) the chamber arrangement, (b) the multilayer structure of tentacle element

When one chamber is pressured, the chamber will be bent on opposite direction. Each segment can be bent in any direction in the plane, by appropriate control of pressures of the two chambers. The electro-rheological fluid is electrically controlled by means of two electrodes mounted of each chamber. A high voltage [6, 10] kV is applied on the central electrode and the peripheral electrodes are linked to the ground. The chambers construction is a multilayer one, and the arm becomes inflexible when it is applied an electric field [9, 10].

In comparison with the conventional systems, the presented structure has a number of features, such as:
- the stability of the system can be easily improved by electro-rheological fluid viscosity control;
- a controller for the ER fluid control can be developed and the complex control algorithms can be implemented;
- the presented system has a great power density;
- the system has many degree of freedom.

2. Dynamic model of the planar tentacle arm

In order to develop the dynamic model of a planar hyper-redundant robot [5], a certain segment of the arm, which is noted with *i*, will be taken. The equivalent mass of the superior segments is noted with *M*, as we seen in figure 3. It is supposed that the tentacle segment can operates in the vertical plan xOz only, and the segment has the uniform distributed mass with a linear density ρ, where ρ represents the equivalent density of the composite materials and the electro-rheological fluid.

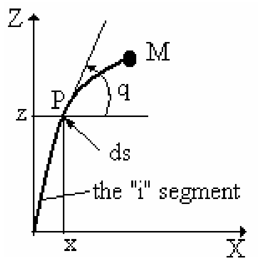


Figure 3. The representation of the *i* segment of arm

It will be noted with *s* the spatial variable of segment length, $s \in [0, L]$. The position of one point *P* from the robot arm will be written in following form

$$x = \int_0^s \cos q' \cdot ds', \tag{1}$$

$$z = \int_0^s \sin q' \cdot ds', \tag{2}$$

where *q* is the Lagrange generalized coordinate, in this case being the absolute angle $s' \in [0, s]$, and $q' = q(s')$.

The generalized coordinate for the system that is represented in figure 3 will be a temporal function

$$q = q(s,t). \tag{3}$$

The dynamic behaviour of the hyper-redundant arm is determined by the effects of torque which is distributed along the arm and of the system energy. The distributed torque is symbolized with $T(s)$ and it is given by the difference of the pressures from the two chambers of each segment [6, 7].

The velocities components in relation to *x* and *z* axes will be considered, that is symbolized v_x and v_z. These velocities are obtained by derivation the relations (1) and (2).

$$v_x = \dot{x} = \int_0^s (-\sin q') \cdot \dot{q}' \cdot ds', \tag{4}$$

$$v_z = \dot{z} = \int_0^s (\cos q') \cdot \dot{q}' \cdot ds' \cdot \tag{5}$$

By considering an element with the elementary mass *dm*, the kinetic energy of the segment will be

$$dE_{c1} = \frac{v^2}{2} \cdot dm = \frac{v_x^2 + v_z^2}{2} \cdot dm. \tag{6}$$

Taking account the velocities expressions from (4) and (5), and $dm = \rho \cdot A \cdot ds$, the relation (6) becomes

$$E_{c1} = \frac{1}{2}\int_0^L \rho \cdot A \cdot \left[\left(\int_0^s (\sin q') \cdot \dot{q}' \cdot ds' \right)^2 + \left(\int_0^s (\cos q') \cdot \dot{q}' \cdot ds' \right)^2 \right] \cdot ds \tag{7}$$

The kinetic energy of superior segments of tentacle structure, that have the equivalent mass *M*, is obtained analogously, such as follow

$$E_{c2} = \frac{1}{2} M \left[\left(\int_0^L (\sin q') \cdot \dot{q}' \cdot ds' \right)^2 + \left(\int_0^L (\cos q') \cdot \dot{q}' \cdot ds' \right)^2 \right] \tag{8}$$

The elementary potential energy for the distributed mass of segment will be as follows,

$$dE_{p1} = g \cdot z \cdot dm, \tag{9}$$

and the entire potential energy will be

$$E_{p1} = \rho \cdot A \cdot \int_0^L \int_0^s \sin q' \cdot ds' \cdot ds. \tag{10}$$

For the superior segments of the hyper-redundant arm, the potential energy will be the integral form

$$E_{p2} = M \cdot g \cdot \int_0^L \sin q \cdot ds. \tag{11}$$

The deformation energy of the segment [5] is in the following form

$$E_{cL} = \int_0^L \frac{D^2}{2} E \cdot q^2 ds, \tag{12}$$

where D is the cylinder diameter, and E is the elastic modulus of the material.

The energy that correspond to the fluid viscosity [8, 9, 10] has the following form

$$E_v = \int_0^L \eta \cdot q \cdot \dot{q} \cdot ds \,. \tag{13}$$

In order to obtain the dynamic behaviour of the tentacle arm, the Lagrange method for infinite dimensional systems [7] will be used,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}(s)}\right) - \frac{\delta L}{\delta q(s)} = T(s), \tag{14}$$

where L is the Lagrangian function,

$$L = E_c + E_{nc}\,, \tag{15}$$

where E_c and E_{nc} represent the mechanical work of the conservative forces and the mechanical work of the non-conservative forces, respectively.

In the relation (2.14), $\delta L/\delta q$ means the functional partial derivative [2], which is definite as the variation of the L Lagrangian function in relation to the q function in a point $s \in [0, L]$, and T is the generalized input of the system. By replacing the relations (7) – (13) in (14) and computing the partial derivative, the general form of the arm dynamics is as follows

$$\begin{aligned} &\rho A \int_0^s \left(\sin(q - q')\dot{q}'^2 + \cos(q - q')\ddot{q}'\right) ds' + \\ &+ M \int_0^L \left(\sin(q_L - q')\dot{q}'^2 + \cos(q_L - q')\ddot{q}'\right) ds' + \\ &+ \rho A \int_0^L \cos q ds + Mg \cos q + Er^2 q + k_v \dot{q} = T \end{aligned} \tag{16}$$

where the following notations are made

$$\begin{cases} q' = q(s',t) \\ q = q(s,t) \\ q_L = q(L,t) \\ s' \in [0, \quad s] \\ s \in [0, \quad L] \end{cases} \tag{17}$$

3. Discrete model

In order to obtain a simplified discrete planar model due to the relation (16), it will be used a spatial discretization $s_1, s_2, \ldots, s_n$ with $s_i - s_{i-1} = \Delta$ and the constraints

$$\begin{cases} \left|q(s) - q(s')\right| < \varepsilon_1 \\ \left|q_L(s) - q(s')\right| < \varepsilon_2 \end{cases} \tag{18}$$

where ε_1 and ε_2 are the small enough constants.

From the relation (14), is obtain

$$\ddot{q} + B\dot{q} + Cq + D = FT \tag{19}$$

where the coefficients B, C, F represent the $n \times n$ matrices, and D is a $n \times 1$ nonlinear vector,

$$B = k_v P^{-1} \tag{20}$$

$$C = Er^2 P^{-1} \tag{21}$$

$$D = P^{-1} Q \tag{22}$$

$$F = P^{-1} \tag{23}$$

$$P = \begin{bmatrix} (\rho A + M)\Delta & M\Delta & \ldots & M\Delta \\ (\rho A + M)\Delta & (\rho A + M)\Delta & \cdots & M\Delta \\ \vdots & \vdots & \vdots & \vdots \\ (\rho A + M)\Delta & (\rho A + M)\Delta & \cdots & (\rho A + M)\Delta \end{bmatrix} \tag{24}$$

$$Q = \begin{bmatrix} Mg \cos q_1 + \rho A \Delta \sum_{i=1}^{n} \cos q_i \\ Mg \cos q_2 + \rho A \Delta \sum_{i=1}^{n} \cos q_i \\ \vdots \\ Mg \cos q_n + \rho A \Delta \sum_{i=1}^{n} \cos q_i \end{bmatrix} \tag{25}$$

$$q_i = q(s_i) \tag{26}$$

$$q = [q_1, \quad q_2, \quad \ldots, \quad q_n]^T \tag{27}$$

$$T_i = T(s_i) \tag{28}$$

$$T = [T_1, \quad T_2, \quad \ldots, \quad T_n]^T \tag{29}$$

A linear model can be obtained if we neglect the terms which depend on gravity force. The relation (19) can be write in the following simplified manner

$$\ddot{q} + B\dot{q} + Cq = FT \tag{30}$$

where B, C are the matrices with variable coefficients that are determined by the electro-rheological fluid viscosity and by electric filed intensity.

From the relation (19) it can obtained classical model by using the state vector,

$$x = [q, \quad \dot{q}]^T \tag{31}$$

and the matrices

$$H = \begin{bmatrix} O & I \\ -C & -B \end{bmatrix}, \; g = \begin{bmatrix} O \\ -D \end{bmatrix}, \; G = \begin{bmatrix} O \\ F \end{bmatrix} \tag{32}$$

The state equations will be

$$\dot{x} = Hx + g(x) + GT \tag{33}$$

where $g(x)$ contains the nonlinear terms.

4. The nonlinear observer

In order to derive the control law of the tentacle arm, it is necessary to know the state variable $q(s)$, that is distributed on the entire arm length, with $s \in [0, L]$. The values of these parameters are very difficult to be obtained from the information which are provided by the transducers mounted on the robot arm. Thus, it can measure only the variable $q(0) = q_1$, for s = 0 which represent the variable of the arm joint.

In order to estimate the state values $q(s)$, with $s \neq 0$ non-accessible, it must to find a nonlinear observer. It will be supposed that only the state variable $q(0) = x_1$ can be measured. In order to develop a nonlinear observer model, the discrete nonlinear model of the tentacle robot (33) and the output

$$y = v^T x , \tag{34}$$

where v is a such vector

$$v = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \end{bmatrix}^T \tag{35}$$

will be used.

Theorem. The nonlinear observer is defined by the relation

$$\dot{z} = Rz + h(z) + ST + Ky \tag{36}$$

where $z \in \Re^{2n}$, R, S, K are constant matrices of dimensions $(2n\times 2n)$, $(2n\times 2n)$ and $(2n\times 1)$, respectively, and the vector $h(z)$ is a $(2n\times 1)$ nonlinear vector.

The observer elements must verify the following conditions

$$z = \Upsilon x \tag{37}$$

$$R\Upsilon + KV^T = \Upsilon H \tag{38}$$

$$h(\Upsilon x) = \Upsilon g(x) \tag{39}$$

$$S = \Upsilon G \tag{40}$$

Prove

In the case of the “sliding mode” method, the control system is proved by Lyapunov technique [10]. Thus, a V Lyapunov function is defined such as

$$V = \frac{1}{2}\sigma^2(e,\dot{e}) \tag{41}$$

where V is a continuous and differentiable function with $V(e,\dot{e}) \geq 0$, and $V(e,\dot{e}) = 0$ if and only if $\sigma(e,\dot{e}) = 0$, where function $\sigma(e,\dot{e})$ is as follows

$$\sigma(e,\dot{e}) = me + \dot{e} = 0 \tag{42}$$

The next condition is imposed

$$\dot{V}(e,\dot{e}) \leq 0 \tag{43}$$

and taking into account (41) and (42), it will obtain

$$\begin{aligned}\dot{V}(e,\dot{e}) &= \sigma(e,\dot{e})\cdot\dot{\sigma}(e,\dot{e}) = \\ &= \sigma(e,\dot{e})\cdot(m\cdot\dot{e} + \ddot{e}) \leq 0\end{aligned} \tag{44}$$

A reference input is considered, noted with $r(t)$ and smooth enough,

$$r(t) = e(t) + q(t) \tag{45}$$

that checks the following constraints

$$\begin{cases} |r(t)| < \alpha_1 \\ |\dot{r}(t)| < \beta_1 \\ \ddot{r}(t) = 0 \end{cases} \tag{46}$$

Considering $n = 1$, it will obtain

$$\begin{aligned}&\dot{V}(e,\dot{e}) = \\ &= \sigma(e,\dot{e})\left[m\cdot\dot{e} - \frac{1}{\delta}\left(T - \eta(\dot{r} - \dot{e}) - c(r - e)\right)\right]\end{aligned} \tag{47}$$

where the coefficients δ and c are as follows

$$\delta = (\rho\cdot A + M)\Delta,\ c = E\cdot\frac{d^2}{4} \tag{48}$$

The equation (47) can be rewritten as

$$\begin{aligned}&\dot{V}(e,\dot{e}) \leq \frac{1}{\delta}\sigma(e,\dot{e})T + \\ &+ |\sigma(e,\dot{e})|\left[|m\cdot\dot{e}| + \frac{1}{\delta}|\eta(\dot{r} - \dot{e})| + \frac{c}{\delta}|r - e|\right]\end{aligned} \tag{49}$$

The final form of the expression from (49) is

$$\begin{aligned}&\dot{V}(e,\dot{e}) \leq \frac{1}{\delta}\left(\sigma(e,\dot{e})T + |\sigma(e,\dot{e})|\left[|\dot{e}|\left(m + \frac{\eta}{\delta}\right) + \right.\right. \\ &\left.\left. + |e|\frac{c}{\delta} + \left(\frac{\eta}{\delta}\beta_1 + \frac{c}{\delta}\alpha_1\right)\right]\right)\end{aligned} \tag{50}$$

The control law can be selected as

$$\begin{aligned}T = &-(k_1\cdot\operatorname{sgn} e + k_2\cdot\operatorname{sgn}(\sigma\cdot\dot{e})\cdot\dot{e} + \\ &+ k_3\cdot\operatorname{sgn}(\sigma\cdot e)\cdot e\end{aligned} \tag{51}$$

By replacing (37) into (50), it results

$$\begin{aligned}&\dot{V}(e,\dot{e}) \leq \frac{1}{\delta}(-k_1 + \eta\cdot\beta_1 + c\cdot\alpha_1)|\sigma| + \\ &+ \frac{1}{\delta}(-k_3 + c)|\sigma\cdot e| + \left(-\frac{k_2}{\delta} + m + \frac{\eta}{\delta}\right)|\sigma\cdot\dot{e}|\end{aligned} \tag{52}$$

If the controller amplification coefficients k_1, k_2, k_3 verify the constraints

$$\begin{cases} k_1 > \eta\cdot\beta_1 + c\cdot\alpha_1 \\ k_2 > \delta\left(m + \dfrac{\eta}{\delta}\right) \\ k_3 > c \end{cases} \tag{53}$$

the $\dot{V}(e,\dot{e})$ will be negative, and the relation (52) can be rewritten as

$$\dot{V}(e,\dot{e}) \le \frac{1}{\delta}\left(-k_1^* + \eta\cdot\beta_1 + c\cdot\alpha_1\right)\left|\sigma(e,\dot{e})\right| = \\ = -\gamma\left|\sigma(e,\dot{e})\right| \quad (54)$$

where the k_1^* factor verifies the relation (52).

The commutation time [10] can be derived as follows

$$t_s = \frac{1}{\gamma}\left|\sigma(e(0),\dot{e}(0))\right| \quad (55)$$

5. Simulation results

In the follows, several results by simulation are presented. A hyper-redundant arm with two elements is considered. The main parameters of the robot are presented in Table 1. The manipulator performances are tested with a variable structure controller.

Table 1. The tentacle robot parameters with 2 segments

Segment	Length [m]	Diameter [m]	ω_n	ξ_{init}
1	0.12	0.014	110	0.7
2	0.08	0.010	80	0.65

First, the nonlinear observer is verified. The observer convergence is easily obtained for this case $(n=2)$, and the model matrices are

$$R = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 1.01 & -1 & 0 & -1 \\ 1.48 & 0 & -1 & -1 \\ 1.15 & 0 & 0 & -1 \end{bmatrix},\ K = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \quad (56)$$

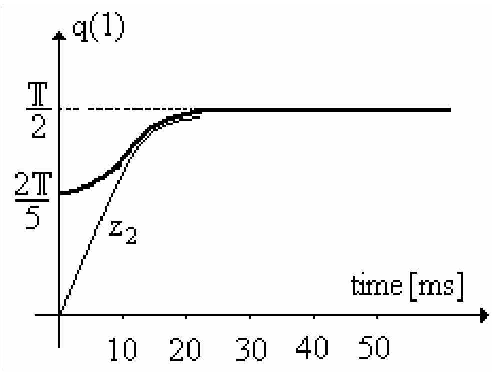


Figure 4. The movement performed by robot between 2π/5 and π/2

The movement of the first tentacle segment, respectively $q(1)$, was represented between $2\pi/5$ and $\pi/2$ in the figure 4. The estimated values for the observer in the following conditions was represented with full line,

$$\begin{cases} z_2(0) = q^0(1) = 0 \\ z_4(0) = q^0(1) = 0 \end{cases} \quad (57)$$

Then, "bang – bang" controller

$$T = -\left[k_1\cdot\mathrm{sgn}(e) + k_2\cdot\mathrm{sgn}(\sigma\cdot\dot{e})\dot{e} + \\ + k_3\cdot\mathrm{sgn}(\sigma\cdot e)e\right] \quad (58)$$

is tested.

A switching line for $m = 2$ is chosen. The evolutions of the control law u and of the $q(1)$ coordinate are represented in figure 5.

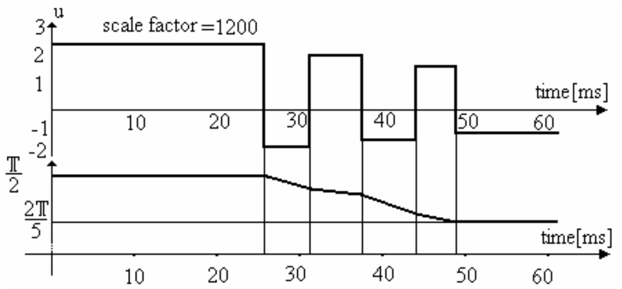


Figure 5. The evolutions of the control law *u* and of the *q*(1) coordinate

Next, a direct control law is used. The initial conditions of the proposed trajectory are

$$\begin{cases} q_1(0) = 2\pi/5 \\ \dot{q}_1(0) = 0 \\ q_2(0) = 2\pi/5 \\ \dot{q}_2(0) = 0 \end{cases} \quad (59)$$

The switching times have the values

$$\begin{cases} t_{s1} \le 0.026\,s \\ t_{s2} \le 0.017\,s \end{cases} \quad (60)$$

The simulation of the control system for two different controls is shown in figure 6. The curve 1 (figure 6) represents the system trajectory for a step input in absence of a variable structure control. The second curve from figure 4 contains two domains. The first domain is characterized by damping coefficient ξ = 0.45, and the second domain has a damping coefficient ξ = 1.45.

6. Conclusion

In this work, namely the hyper-redundant arm with continuum elements was treated. First, the discrete dynamic model of the tentacle arm was analyzed. The dynamic model of the tentacle arm with continuum elements, based on energetic relations, in conjunction with the electro-rheological fluid was discussed. Then, the discrete planar model based to the nonlinear observer was developed. Also, numerical simulations of the arm motion proved the correctitude the solutions.

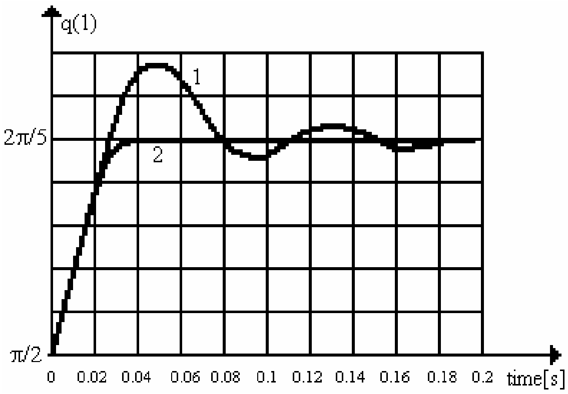


Figure 6. The system response to a input step for different control laws: (1) control without variable structure, (2) control with variable structure

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