Abstract. Predictive maintenance is used to improve the availability of industrial bodies while minimizing maintenance costs. Monitoring the evolution of defects and early detection are based on periodic controls to avoid failures and to plan maintenance operations considered relevant which aims to guarantee normal operation later. So talking about the future is knowing the present.

In this context, a stochastic model is discussed based on a type of Markov chain \( X_n, n = 1, 2, 3 \). That is, if \( X_n = i \) then the process is said to state \( i \) at time \( n \) with the existence of a probability \( P_{ij} \) that this process will be in State \( j \) at time \( (n + 1) \). Thus, the conditional distribution of a future state of \( X_{n+1} \) is independent of the past but only the \( X_n \) present state-dependent States.

In this study, is presented the methodology to model and simulate the evolution of the degradation occurring in mechanical parts, the cases studied its double row bearings installed on centrifugal fan of average power. Each bearing, we propose to associate a Markov chain describing the transition from one State \( i \) to another \( j \) State following the evolution of the overall level of vibration, knowing that according to standards (NF, ISO, VDI, API...) the degradation process of imports it what mechanical body passes by States such as: 'Good', 'Eligible', 'Yet eligible' and 'Ineligible' or 'Disentitled'.

The proposed model to calculate the probability of having a bearing faded along life. In addition, it is possible to use the transition matrix associated with Markov chain analysis of operation for the determination of the mechanical system reliability.

Keywords: Conditional maintenance, Vibration, Modelling Markov chain

1. Introduction

The establishment of a model for estimating the State has considerable importance in maintenance. Predictive maintenance, therefore, has become in recent years a key objective of the industry. Deemed necessary, see even indispensable, by the high-tech sectors [1], and became the concerns of small and medium-sized enterprises to optimize production, increase the safety of the staff and reduce the regular preventive or corrective maintenance costs.

This work focuses on the estimation of the real state of a mechanical body problem (the case studied is a bearing). It starts from the observation that the knowledge of the State is one of the imperatives of any decision-making base, knowing that when the piece design study or an industrial body, it is essential to take into account different states by which this piece passes during its degradation and is likely to suffer throughout his life as well during 'normal' operation that's unexpectedly (as a result overloading, vibration, shock...).

For this purpose, it must adapt tools to estimate, reliable and well manage information based on vibration measurements [2]. This function depends on the different phases or States of the degradation process of a body mechanical namely the state 'Good', 'Eligible', 'Yet eligible', the State 'Ineligible' from one side and the vibration amplitude and the size of this piece of other side. In our study, we have based on the analysis of the overall level of vibration that is designed to describe the overall condition of a body in comparison to standards (standard) or measures previous (evolutionary analysis based on historical).

The frequency of the vibration measures is adapted depending on the evolution of the indicators. More increase is rapid, more controls should be reconciled. It is mandatory that the operating conditions of the machine as well as the conditions of measurement (speed, load, temperature etc.) must be precisely the same measure in the other.

Quantification of vibratory thresholds [3] are very tricky and depends on many parameters, such as mounting (media, levels, alignment...), tolerance of the constructor, lived the machine, the user needs... However, orders of magnitude limiting States mentioned above are specified in the standards (NF, ISO, VDI, API ...). These standards (Figure 1) must be taken only as suggestions, not as
an absolute reference. At the beginning of monitoring thresholds may refer to standards, but thresholds will be set only after return to the experience.

2. Hypotheses

The assumptions taken into consideration are the following:
- The vibration measurements are taken under the same conditions;
- Bearings in invalid state (Advanced degradation) are followed directly by a change operation;
- Are programmed maintenance operations to reduce the level of vibration than for bearings with States 'entitled' and 'still entitled';
- The vibrations recorded during the break-in period are neglected.

3. Methodology

The study is also being considered for the follow-up of the degradation of double-row roller bearings. This allows, among other things, to evaluate duration limit of proper functioning and evolution of the bearing state change to anticipate change. In many situations, the degradation of a bearing is manifested by a beginning of chipping, for what it is called programmed vibration at different time intervals regular measurement called 'age class'.

The evolution of the degradation of a bearing [4] is expressed by a passage of a 'Good' State to another "degraded or Ineligible" passing through intermediate States 'eligible' and 'disentitled'. These states are given by an integer that describes the number of considered states: 1, 2, ..., n (our cases n = 4).

In this perspective, is considered each degradation necessarily by these States, so a growing suite of States, along the lines of the degradation with consideration of servicing performed during a change of State actions, is observed.

For example, if a bearing is the State 'degraded or Ineligible' at time (t), it may be a 'good' State at the moment (t + 1) and this all depends on carried out maintenance operations, but if it's a 'disentitled' state change will be directly programmed and this under the hypothesis (2).

The objective of this study is to associate a mathematical model (transition matrix Markov) to the results obtained during a statistical study done on the degradation of a batch of 60 bearing, this bearing type is installed on centrifugal fans at the level of a complex steel and who work in identical conditions for a mate of vibration measurement of 30 months (t = 0, ..., 30) with a periodicity of measurement of two months, so is before 15 vibration measure data are represented by the Table 1.

4. The model presentation

A Markov chain is a sequence of random variables \(X_n, n \in \mathbb{N}\) that allows modeling the dynamic evolution of a random system: \(X_n\) represents the State of the system at the moment n. The fundamental property of Markov chains [5], known as Markov property, is that its future depends on the past through its current value. That is, conditionally \(X_n, (X_0, ..., X_n)\) and \((X_{n+k}, k \in \mathbb{N})\) are independent. The applications of Markov chains are very numerous (networks, genetic populations, mathematics financial management stock, stochastic optimization algorithms, simulation ...).

The law of 'transition probability' is defined by a State \(i\) to State \(j\) by:

\[
P_{ij} = P(X_n = j | X_{n-1} = i).
\]
Each bearing (or each piece), will be associated to a Markov chain that reflects its degradation; it is possible to present the parameters of the model by its graphic form that expresses the various States and their relationship (figure 2). Each bearing, during its lifetime, spent by four States. It draws so four circles symbolizing each event States: A: Good, B: Eligible, C: Yet eligible and D: Unacceptable (Ineligible).

![Figure 2. Markov model under graphic shape](image)

The transition matrix is represented as follows:

$$\mathbf{M} = \begin{bmatrix}
P_{aa} & P_{ab} & P_{ac} & P_{ad} \\
P_{ba} & P_{bb} & P_{bc} & P_{bd} \\
P_{ca} & P_{cb} & P_{cc} & P_{cd} \\
P_{da} & P_{db} & P_{dc} & P_{dd}
\end{bmatrix}. \quad (2)
$$

The bearings state change is a Markov process given by the above transition matrix. Knowing that a Markov transition matrix is characterized by:

- \((P_{ij})\) coefficients are always less than or equal to 1;
- The sum of the coefficients of the same line is equal 1.

In the end, to calculate the bearings degradation probability based on the Markov model, simply calculate the power of the transition matrix for each age class, i.e. for age class \((n)\) the corresponding probability will be calculated in the following way:

So general, for \(n\) measurement (age class) with \((n = 1, 2, 3, ..., 15)\), the model describing the bearings State change is given by:

$$X^{(1)} = X^{(0)} \cdot \mathbf{M} \quad (3)$$

or

$$X^{(n)} = X^{(n-1)} \cdot \mathbf{M} \quad (4)$$

with:

- \(X^{(0)}\) is the line matrix that represents the probabilities of the bearings States for a measure vibration (or age class) 0;
- \(X^{(n)}\) is the line matrix that represents the probabilities of the bearing States for vibration measurement \((n - 1)\);
- \(\mathbf{M}\) is the Markov transition matrix.

5. Discussion of the results

According the data shown in Table 2, middle passage probabilities of a good state for another gradient through intermediate states are respectively: 0.56; 0.11; 0.12 and 0.21.

Table 2. The probability of bearings change state

<table>
<thead>
<tr>
<th>Cumulative number of degraded bearings</th>
<th>Degradation probability</th>
<th>Survival probability</th>
<th>(P_{(1)})</th>
<th>(P_{(2)})</th>
<th>(P_{(3)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>1.00</td>
<td>0.03</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.95</td>
<td>0.13</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>0.85</td>
<td>0.09</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>0.83</td>
<td>0.14</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.22</td>
<td>0.78</td>
<td>0.08</td>
<td>0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>6</td>
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<td>0.72</td>
<td>0.11</td>
<td>0.06</td>
<td>0.05</td>
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<tr>
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<td>0.62</td>
<td>0.05</td>
<td>0.23</td>
<td>0.14</td>
</tr>
<tr>
<td>8</td>
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<td>0.48</td>
<td>0.11</td>
<td>0.22</td>
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<tr>
<td>9</td>
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<td>0.07</td>
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<td>0.25</td>
<td>0.09</td>
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<td>0.22</td>
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<tr>
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<td>0.15</td>
<td>0.33</td>
<td>0.09</td>
<td>0.46</td>
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<tr>
<td>12</td>
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<td>0.15</td>
<td>0.33</td>
<td>0.23</td>
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<tr>
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<td>0.07</td>
<td>0.11</td>
<td>0.53</td>
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<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.09</td>
<td>1.00</td>
</tr>
</tbody>
</table>

with:

- \(P_{(1)}\): Probability of 'eligible' to time State \((t)\) knowing that it is the 'Good' State at the time \((t - 1)\);
- \(P_{(2)}\): 'yet eligible' State probability at time \((t)\) knowing that he is 'eligible' to the time \((t - 1)\);
- \(P_{(3)}\): 'Degraded or Ineligible' probability at the time State \((t)\) knowing that it is 'yet eligible' time \((t - 1)\).

The matrix of transition could be written in the form below, knowing that the sum of the coefficients of each line for a Markov matrix is equal to 1:
Consider the hypothesis that a bearing is considered good at the end of the first age class.

\[ X^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \]

At the beginning of the second age class, is predicted:

\[ X^{(1)} = X^{(0)} \cdot M = M_1 \]

\[ M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.56 & 0.11 & 0.12 & 0.21 \\ 0 & 0.15 & 0.50 & 0.35 \\ 0 & 0.13 & 0.17 & 0.70 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.56 & 0.11 & 0.12 & 0.21 \end{bmatrix} \]

Thus, after the first measurement of vibration (corresponds to the first age class) on a 56% chance that the bearing is in good condition, 11% in eligible State, 12% yet eligible and 21% in inadmissible or degraded state.

After a period of service (represented by a well-defined age class (n), our model will be given by:

\[ X^{(n)} = X^{(0)} \cdot M^n = M_n \]

\[ M_n = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.56 & 0.11 & 0.12 & 0.21 \\ 0 & 0.15 & 0.50 & 0.35 \\ 0 & 0.13 & 0.17 & 0.70 \\ 0 & 0 & 0 & 1 \end{bmatrix}^n \]

And so on, for the last companion of vibration measurement which corresponds to 30 months of service (or 15 age class), you get:

\[ X^{(n)} = X^{(0)} \cdot M^{15} = M_{15} \]

\[ M_{15} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.56 & 0.11 & 0.12 & 0.21 \\ 0 & 0.15 & 0.50 & 0.35 \\ 0 & 0.13 & 0.17 & 0.70 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{15} = \begin{bmatrix} 0.0001 & 0.0001 & 0.0001 & 0.9995 \end{bmatrix} \]

According the model proposed, the probability to track the State of a bearing change based on the age-class is given in Figure 3.

The results show clearly that the proposed model provides a good estimate of the probability of the state change and the passage of a good state to another state of a bearing. There is also the functions of these probabilities plotted according to the model (Figure 4 a and b) wears the same allure that calculated by methods of classical statistics, also based on the correlation coefficient value respectably (r = 0.77; 0.85). The results calculated by the application of the proposed model are very close to reality.

Proposed model, therefore, provides information sought to find out about the real state of the bearing.

6. Conclusions

The decision to change a bearing is made according to the vibratory threshold reached. A bearing will never be changed when the levels is still acceptable. It is necessary to avoid exceeding the threshold of danger, because starting from this threshold, the bearing can break constantly. For that the use of the vibratory analysis systematically makes it possible to make an assessment on the real state of the bearing of equipment. In the event of defect, it is an invaluable help to identify and evaluate the failure of it.

The presented study shows that the Markov model proposed for the monitoring of the evolution of the degradation of a rolling bearing lubricated for life accurately describes its actual state and provides a good prediction of failure.

This pattern should also be able to assist in the planning of the operation of predictive maintenance and to avoid the unexpected. These results tell us about the actual States of the studied bearing probabilities. This paves the way for a study generalized on other types of bearings with taking account of preventive operations improvements in their functioning.
Figure 4. Probability of after the model proposed and calculated probability
(a): Ineligible or degraded state; (b): Good state

References

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