Fractals and Fractal Design in Architecture

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Abstract
Fractal geometry defines rough or fragmented geometric shapes that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. In short, irregular details or patterns are repeated themselves in even smaller scale. Fractal geometry deal with the concept of self-similarity and roughness in the nature.

The most important properties of fractals are repeating formations, self-similarity, a non-integer dimension, and so called fractional size which can be defined by a parameter in irregular shapes. Fractals are formed by a repetition of patterns, shapes or a mathematical equation. Formation is dependent on the initial format.

Not only in nature, fractals are also seen in the study of various disciplines such as physics, mathematics, economics, medicine and architecture. For a variety of reasons, in different cultures and geography, many times the fractal pattern had reflected on creating the architecture.

In the computer-aided architectural design area, fractals are considered as a subset for the representation of knowledge for design aid and syntactic science of the grammatical form. If compared with the grammar of shapes, the number of rules used in the production process of fractals is defined as less, with number of repetitions as more and self-similarity feature, it can be a tool to help qualified geometric design.

A simple form produced with fractal geometry with ultimate repetition is being transformed into an algorithmic complex. This algorithm with an initial state and a production standard that applies to this initial state produces self-similar formats.

In this study, the development of the fractals from the past to the present, the use of fractals in different research areas and the investigation of examples of fractal properties in the field of architecture has been researched.

Keywords
fractals, fractal geometry, fractal design, fractals in architecture

1. Introduction
The term "fractal" was coined by Benoit Mandelbrot to describe the geometry of the highly fragmented forms of nature that were perceived as amorphous (such as trees and clouds), and not easily represented in Euclidean geometry. A second aspect of fractal geometry is its recognition of the practically infinite number of distinct scales exhibited by units of length in patterns in nature. Thirdly, fractals involve chance, their regularities and irregularities being statistical. Finally, they engage the Hausdorff-Besicovitch dimension, an effective measure of complexity, scalar diversity of fragmentation [1]. The mathematical definition of fractal is a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension.

There are many relationships between architecture, arts and mathematics for example the symmetry, the platonic solids, the polyhedral, the golden ratio, the spirals, the Fibonacci’s sequence, but it is difficult to find some interconnections between fractals and architecture. This paper investigates some relationships between architecture and fractal theory [2].

As the following pages indicate, fractal geometry, in opposition to Euclidean geometry, offers better methods for description or for producing similar natural-like objects respectively. Fractals can be found everywhere from coastlines, border-lines and other natural rough lines to clouds, mountains, trees, plants and maybe also in architecture. The following chapters explain what a fractal is in general and
how fractals can be used for architectural analysis and in the stage of planning. The aim of this work is to present how the fractal geometry is helping to newly define a new architectural models and an aesthetic that has always lain beneath the changing artistic ideas of different periods, schools and cultures.

2. What is Fractal?

The best way to define a fractal is through its attributes: a fractal is ‘rugged’, which means that it is nowhere smooth, it is ‘self-similar’, which means that parts look like the whole, it is ‘developed through iterations’, which means that a transformation is repeatedly applied and it is ‘dependent on the starting conditions’. Another characteristic is that a fractal is ‘complex’, but it can be described by simple algorithms – that also means that beneath most natural rugged objects there is some order [3]. The term ‘fractal’ comes from the Latin word ‘fractus’ which means ‘broken’ or ‘irregular’ or ‘unsmooth’ as introduced by Benoit Mandelbrot at 1976 [4]. Mandelbrot coined the term ‘fractal’ or ‘fractal set’ to collect together examples of a mathematical idea and apply it to the description of natural phenomena as the fern leaf, clouds, coastline, branching of a tree, branching of blood vessels, etc. (Fig. 1).

![Fractals in nature](image)

2.1. Fractal dimension

Fractals can be constructed through limits of iterative schemes involving generators of iterative functions on metric spaces. Iterated Function System (IFS) is the most common, general and powerful mathematical tool that can be used to generate fractals [6].

Mandelbrot proposed a simple but radical way to qualify fractal geometry through a fractal dimension. The fractal dimension is a statistical quantity that gives an indication of how completely a fractal appears to fill space, as one zooms down to finer scales. This definition is a simplification of the Hausdorff dimension that Mandelbrot used as a basis. This gives an indication of how completely a particular fractal appears to fill space as the microscope zooms in to finer and finest scales. Another key concept in fractal geometry is self-similarity, the same shapes and patterns to be found at successively smaller scales [1]. There are two main approaches to generating a fractal structure: growing it recursively from a unit structure, or constructing divisions in the successively smaller units of the subdivided starting shape, such as Sierpinski’s triangle.

However, it should be noted that there are many specific definitions of fractal dimensions, such as Hausdorff dimension, Rényi dimensions, box-counting dimension and correlation dimension, etc., and none of them should be treated as the universal one. Practically, the fractal dimension can only be used in the case where irregularities to be measured are in the continuous form [6].

2.2. The Appearance of Fractals in the History

The mathematical history of fractals began with mathematician Karl Weierstrass in 1872 who introduced a Weierstrass function which is continuous everywhere but differentiable nowhere. In 1904, Helge von Koch refined the definition of the Weierstrass function and gave a more geometric definition of a similar function, which is now called the Koch snowflake. In 1915, Waclaw Sierpinski constructed self-similar patterns and the functions that generate them. George Cantor also gave an example of a self-similar fractal. In the late 19th and early 20th, fractals were put further by Henri Poincare, Felix Klein, Pierre Fatou and Gaston Julia. In 1975, Mandelbrot brought these work together and named it ‘fractal’ [6]. He defined a fractal to be “any curve or surface that is independent of scale”. This property referred
to as self-similarity, means that any portion of the curve if blown up in scale would appear identical to the whole curve. Then the transition from one scale to another can be represented as iterations of a scale process. Prior to Mandelbrot there were a few contributions in this field by lots of other renowned mathematicians and scientists, but they remained scattered. Some of the theories are chronologically listed below to give an idea of the delighted interest mathematicians showed towards the complex nature of fractals [7].

2.2.1. Cantor’s comb (1872)
George Cantor (1845-1918) evolved his fractal from the theory of sets. All the real numbers in the interval [0, 1] of the real line is considered. The interval (1/3, 2/3) which constitutes the central third of the original interval is extracted, leaving the two closed (0, 1/3) and (2/3, 1). This process of extracting the central third of any interval that remains is continued ad infinitum. The infinite series corresponding to the length of the extracted sections form a simple geometric progression \[1 + (2/3) + (2/3)^2 + (2/3)^3 + \ldots\] / 3. This shows that this sums to unity, meaning that the points remaining in the Cantor set, although infinite in number, are crammed into a total length of magnitude zero [7] (Fig. 2).

![Fig. 2. Cantor’s comb](image)

2.2.2. Helge von Koch’s curve (1904)
The curve generated by Helge von Koch (1870 – 1924) in 1904 is one of the classical fractal objects. The curve is constructed from a line segment of unit length whose central third is extracted and replaced with two lines of length 1/3. This process is continued, with the protrusion of the replacement always on the same side of the curve, to get the Koch’s curve [9] (Fig. 3).

![Fig. 3. Helge von Koch’s curve](image)

2.2.3. Sierpinski’s triangle (1915)
Sierpinski considered a triangle whose mid points where joined and the triangle thus formed extracted. The same process is repeated on the resulting triangles also. When this is repeated ad infinitum we get the Sierpinski’s Triangle, which is a good example of a fractal [9] (Fig. 4).

![Fig. 4. Sierpinski’s triangle](image)
2.2.4. Gaston Julia sets (1917)
Fractals generated from theories of Gaston Julia (1892 – 1978) are based on the complex plane. They are actually a kind of graph on the complex axes, where the x-axis represents the real part and the y-axis represents the imaginary part of the complex number. For each complex number in the plane, a function is performed on that number, and the absolute value of the range is checked. If the result is within a certain range, then the function is performed on it and a new result is checked in a process called iteration [7] (Fig. 5).

![Fig. 5. Gaston Julia set [11]](image)

2.3. Characteristics of fractals
A fractal as a geometric figure or natural object combines the following characteristics [9]:
- Its parts have the same form or structure as the whole, except that they are at a different scale and may be slightly deformed;
- It has a fine structure at arbitrarily small scales;
- Its form is extremely irregular or fragmented, and remains so, whatever the scale of examination;
- It is self–similar (at least approximately);
- It is too irregular to be easily described in traditional Euclidean geometric language;
- It has a dimension which is non–integer and greater than its topological dimension (i.e. the dimension of the space required to "draw" the fractal);
- It has a simple and recursive definition;
- It contains "distinct elements" whose scales are very varied and cover a large range;
- Formation by iteration;
- Fractional dimension.

2.4. Types of fractals
- Natural Fractals
Fractals are found all over nature, spanning a huge range of scales. We find the same patterns again and again, from the tiny branching of our blood vessels and neurons to the branching of trees, lightning bolts, and river networks. Regardless of scale, these patterns are all formed by repeating a simple branching process [9].
- Geometric Fractals
Purely geometric fractals can be made by repeating a simple process. The Sierpinski Triangle is made by repeatedly removing the middle triangle from the prior generation. The number of colored triangles increases by a factor of 3 each step, 1, 3, 9, 27, 81, 243, 729, etc. Another example of geometric fractals is the Koch Curve [9, 12].
- Algebraic (Abstract) Fractals
We can also create fractals by repeatedly calculating a simple equation over and over. Because the equations must be calculated thousands or millions of times, we need computers to explore them. Not coincidentally, the Mandelbrot Set was discovered in 1980, shortly after the invention of the personal computer [9, 12].
- Multifractals
Multifractals are a generalization of a fractals that are not characterized by a single dimension, but rather by a continuous spectrum of dimensions [9].
3. Fractals in Architecture

Architectural forms are handmade and thus very much based in Euclidean geometry, but we can find some fractals components in architecture, too. Fractal geometry has been applied in architecture design widely to investigate fractal structures of cities and successfully in building geometry and design patterns.

Fractal analysis in architecture can be done in two stages:

- Analysis at a small scale (e.g. analysis of a single building) the building’s self-similarity (components of building that repeats itself at different scales) (Fig. 6)
- Analysis at a large scale (e.g. analysis at urban scale) the box counting dimension (to determine the fractal dimension of the building) [4] (Fig. 7).

3.1. Fractal characteristics in the history of architecture

Architecture as a mirror of society is also a kind of public image, which is promoted by our time and by the culture in which we are building. The architect translates and interprets the conscious and unconscious thoughts of society. This also means that the architect has to face history and the present, with fractal geometry belonging to the present and so it should therefore be included in the one or other way.

Fractal building patterns can be traced to ancient Maya settlements. Brown et al. analysed fractal structures of Maya settlements and found that fractals exhibit both within communities and across regions in various ways: at the intra-site, the regional levels and within archaeological sites. Moreover, spatial organization in geometric patterns and order are also fractals, which presents in the size-frequency distribution, the rank-size relation among sites and the geographical clustering of sites [6].

In ‘Fractal Geometry in Architecture and Design’ Carl Bovill measured the fractal dimension of a cubist painting by Le Corbusier and wanted to show by that the lack of interesting detail from a certain scale onwards in modern paintings and buildings. The result of this is that there is a basic rule that leads to general harmony and order. Therefore, the reduction to primary Euclidean shapes and basic colours was an attempt to show the ‘basic natural laws’. But as Bovill’s measurements illustrate, these natural basic laws were only translated into paintings and buildings up to a certain scale [3].

3.1.1. Contemporary usage of fractals

Fractals with its inherent complexity and rhythmic characteristics have also inspired many contemporary architectural design processes. The contemporary usage of fractals in architecture has resulted due to a range of varied concerns. One of the concerns is the organic metaphors of design as used by Peter Eisenman and Zvi Hecker.

Peter Eisenman exhibited his House 11a for the first time in 1978. He adopted a philosophical process of fractal scaling constituting of three destabilizing concepts of: “discontinuity, which confronts the metaphysics of presence, recursively, which confronts origin; and self-similarity, which confronts representation and the aesthetic object [7].

Out of metaphors, the Israeli architect Zvi Hecker developed a ‘land-form’ building, the Jewish school ‘Galinski-Schule’ in Berlin [3]. The overall geometry is taken from a sunflower, which connects snake-shaped corridors, mountain-stairs and fish-shaped rooms. In many examples of similar buildings simple materials, simple forms and abstraction produce complexity like in the Jewish school ‘Galinski-Schule’ in Berlin. This is once more the concept of fractal geometry: to get a complex object out of simple rules or algorithms. This may be the difference to ‘modern-architecture’: complexity in contrast to simplicity.
3.2. Methods of application of fractals in architecture

The application of Fractals in architecture can be usually done in following different methods:

1. Conceptual method: This uses fractal geometry and its concept as a guiding element to its theories. This method provides a theoretical solution that ultimately influences the final form.

2. Geometric-mathematical methods: which uses scheme of counting squares to calculate the Fractal dimension. This method is used to analyse the existing building also.

3. Geometric – intuitive method. This uses the geometry as inspiration for creative expression [4].

3.3. Two-dimensional fractals in architecture

I start off with an overview of two-dimensional fractal forms in architecture, which are mostly present in the ground plans of buildings. This application can be found in a wide range of architectural structures, ranging from the plans of fortifications, to the organization of traditional Ba-ila villages (Zambia) (Figs. 8, 9). The global form of the latter settlements reoccurs in the family ring, which consists of individual houses, which are, again, similar to the overall shape of the village [14]. Most of the ancient African settlements exhibit fractal characteristics. The European settlers found these complicated fractal arrangements as “primitive” when compared to their Euclidean geometry [7]. These intuitive choices of fractal geometry by the ancient settlers show the relevance of fractals with respect to habitat building. Here they practiced an extended family system, which was housed around a ring shaped livestock pen. The pen had a gate at the front and storage houses around it. The buildings became progressively bigger around the ring. A definite status gradient is thus established. The entire settlement is also a ring that constitutes of smaller housing units as described above.

![Figs. 8, 9. Ba-ila settlement village, East Africa [11]](image)

The fractal ground plan that has perhaps received most theoretical attention is Wright’s Palmer House. In order to understand its fractal character, it is important to note that architects sometimes use a ‘module’ as the main organizational element. In a sense, such an element can be understood as the conceptual ‘building block’ of the house (e.g., a circle). Wright often applied this procedure to his work. Initially, the geometry governing his architecture created with the aid of such modules remained Euclidean. In later works, however, these elements were sometimes so organized that they gave the building a remarkable fractal organization. The Palmer House seems to be the culmination point of this evolution. Here, one geometric module – an equilateral triangle – is repeated in the ground plan on no less than seven different scales [14] (Fig. 10).

3.4. Three-dimensional fractals in architecture

A three-dimensional method, which some have linked to fractals, is to tessellate the architectural façade. On first sight, the link with fractal geometry could seem obvious: such patterns are rich in detail, which is an intrinsic characteristic of fractals [14]. The contemporary architectural group Ashton Raggatt McDougall was perhaps one of the first to apply fractal tiling to architecture. They covered the façade and the interior of Storey Hall (Melbourne) with polygon tiles that are inspired on Penrose tiling (Figs. 11, 12).
A 'tiling-approach' has also been adopted by the Lab Architecture Studio for the Federation Square in Melbourne and its adjacent buildings. This design for a public space to mark the centenary of the federation of Australian states (an important step in creating a nation from its parts) is a story that reflects at every level the shift in geometry from top-down to bottom-up, encapsulated by complexity theory and fractal geometry (Figs. 13, 14). The fractal self-similarity of the panels became a vital quality in achieving coherence and difference to the facades. The facades of those buildings that define the public spaces, include an almost iconic representation of self-similarity: 1:2:V5 triangles that combine in fives to create larger triangles and therefore, five of these into the next scale of the same proportion. This is an intellectually graspable and simply constructible motif, that is nevertheless combined in ways that generate relentless difference and absence of repetition across the whole site [1].

The variation between the internal and external patterns and between the elements that link their nodes creates, in the steel square sections that mark these geometrical etchings, a boscage through which light is filtered in a way that is powerfully suggestive of the complexity or organic precedents cited by Benoit Mandelbrot as calling for fractal geometry for their description [1].

Another example of three-dimensional fractals in architecture can be seen in Grand Egyptian Museum in Cairo, Egypt (Fig. 15). In this case, the fractals are applied in the façade of the building. Plateau Edge was designed as a vast, sloping, translucent stone wall, inscribed with fractal patterning. For its subdivision, the wall adopts a fractal described by Waclawsieve (or gasket, triangle). It is a recursive subdivision of triangles by sub-triangles with the new vertices in the center of each edge, in which one component triangle – the central one – is omitted in each generation, or iteration (the holes
The design is dominated by the sightlines from every gallery to the pyramids, whose rigorous geometry informed the design, conception and the adoption of the Sierpinski triangle as the motif for the wall.

'Fractile' is a term coined by the design team for the scaleless, self-similar patterning of the building surface, deploying Ammann aperiodic tiling (Figs. 16, 17, 18) that would tile the planar surfaces infinitely, unceasingly, evenly, without repetition, were it not disrupted by a fractal tile subdivision. Robert Amman's aperiodic tiling was enriched tectonically through the creation of a fractal via selective subdivision [1].

In the Ammann set used, there are three differently shaped tiles (Figs. 16, 17). Each one of those three tiles can subdivide perfectly into copies of the same three tile shapes, scaled down exactly by the Golden Ratio. 'Selective subdivision' means subdividing the tiles in this way, but choosing to subdivide some and not others as the subdivision proceeds. Each time a particular tile of the set of three is created - the 'R' tile - the subdivision is stopped for that tile. This creates a fractal that is especially rich because it, too, is aperiodic, and does not repeat in a way that would allow it to be mapped to itself through translation. This is the geometrical underpinning of the 'fractile' [1].

The fractal subdivision results in variable density of tiles and lines at different locations across the tiling pattern. When the pattern is wound onto the spiral walls, the result is different densities at different locations in the building. By having the unfolded building slide over the fractal tiling pattern, Libeskind was able to choose where these areas of greater density should occur. Finally, how should the graphical representation of the fractal be translated to the fabrication of the physical tiling itself? It was not practical to use different sizes. The answer was to raise the tiles in a relief, in which the varying depths represented the different scales in the fractal [1].
3.5. Sustainability and fractal architecture

The idea of buildings in harmony with nature can be traced back to ancient Egyptians, China, Greeks and Romans. At the beginning of 21st century, the increasing concerns on sustainability oriented on buildings have added new challenges in building architecture design and called for new design responses. As the language of nature, it is, therefore, natural to assume that fractal geometry could play a role in developing new forms of design of sustainable architecture and buildings [6].

Complexity can be said as an integral part of aesthetics and it reflects the surrounding place, and aesthetics again being a part of cultural dynamics. Main aim in a sustainable design should be a disease free and a healthy society through thoughtful activities treating each place as an integrated whole.

Talking about Fractal geometry it is observed that each formation is new and amazing only because fractals are like natural objects that are universally beautiful. Similarly, it is necessary that an architecture should be able to extract the essence of fractal geometry which encourages adapting to the context and time. According to Haggard & Cooper (2006), prototypes of sustainable design as proposed the cultural framework need to have the following antecedent attitude as prototypes of sustainable design like:

1. Reality is a unity that has infinite variety
2. We should have “Scalar integrity” where parts affect the whole and vice versa [4].

Harmony between wholes and parts is possible and desirable. Fractal geometry is based on hierarchical principle, which is an essential element of urban planning. The different scales in an area may be House, Housing colony, Panchayat, Taluk/Block, District, State. When the geometric pattern of the spatial system is achieved with accessibility and piling which means that every space is connected to each other as well as itself it can be considered as a fractal structure [4].

4. Conclusion

Fractals have not gained much importance outside the academic environment. May be the limited resources of architectural practice could be a reason. For long architecture has been dwelling on simple mathematics in arriving at its analytical and proportioning tools (ex. golden sections). The inherent repression towards mathematics by most architects has rarely allowed the designer to venture into the complexities of Mathematics.

This paper has illustratively reviewed the fundamental concepts and properties of fractal geometry theory essential to architecture design, as well as the current state of its applications. Fractal geometry has important implications for buildings. The representative review shows that architecture design is not made to be isolated but to anticipate changes in the environment. Accumulation of technological modernizations, destroying, adapting and many changes have caused the design temporal and spatial diversity and complexity. More specifically, sustainable development in a building can be looked upon as adaptability and flexibility over time when it comes to responding to changing environments. Changing perspectives in geometric framework is mandatory for a shift from industrial culture to one that aims at sustainable designs. The knowledge of Fractals has far-reaching impacts. Being fractal can
definitely not be the sole criteria for judging a building in terms of sustainability. In this context, fractal
geometry theory offers an alternative for sustainable architectural design. This paper provides a bridge
between architectural and fractal geometry theory. Thus, the 'same' or the 'regular' part of the fractal
definition suggests that patterns, rules, and knowledge all repeat, at all scales: this part is sustainable or
constant.

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