CHOICE OF SAMPLE TIME IN DIGITAL PID CONTROLLERS

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Abstract. A generalized type of analogue PID controller is presented in the paper. A digital versions of it based on various methods of discretization are examined and corresponding tuning rules are tested in the optimization manner. The research works are focused on the best choice of sample time as a compromise between the desired control system behavior and the constrained control signal. The main results are connected with the optimal sample times in the hybrid two-rate control systems. New performance criteria for its choice are suggested and the satisfied control is shown on the number of numerical simulations.

Key words: digital PID, optimization tuning, optimal sampling time, two-rate digital control systems

1. Introduction

Most often the digital PID controllers design begins with description of the analog PID main expression, toward which the appropriate method for discretization is applied. Thus a connection between the parameters of the two types controllers, and therefore, between the results from tuning of the discrete and analog control device is created. In this paper a universal algorithm [2, 3, 6] in the form

$$u(s)=K_p\cdot\left\{[b\cdot r(s)-y(s)]+\frac{1}{T_i s}\cdot e(s)+\frac{T_d\cdot s}{1+T_f\cdot s}\cdot[c\cdot r(s)-y(s)]\right\}, \qquad (1)$$

is examined to calculate PID control signal u in an Automatic Control System (ACS) with reference signal r and controlled (output) signal y, where K_p is the gain, T_i and T_d are respectively time constants of integration and differentiation. Parameters b and c (most often $0 \le b, c \le 1$) reflect the degree of influence of the reference signal upon the proportional and differentiation component of the PID controller, respectively. Trough dynamic element with time constant $T_f = T_d / N$ (N = 8 ¸ 20) an additional filtering of the disturbance in ACS is made, which essentially improves the working capacity of the PID controller [1].

In chapter 2 a discrete equivalent of (1) is described, together with additional schematic "anti-windup" mechanism, which correspondence with (1) is analytically proved and confirmed with simulation under different conditions of discretization [1]. A choice of proposed sample time is made by analogy of the research works in [4] and the results over three test plants with considerable uncertain dynamics are shown in chapter 3. Original optimization criteria for optimal choice of sample time [5] are formulated and the degree of coincidence between the signals in the analog- and discrete-analog ACS is analyzed in chapter 4. In chapter 5 two-rate ACS is applied, in which the discrete PID is designed for relatively larger sample time in comparison with the sample time of the whole ACS.

2. Universal description of digital PID controller

The digital equivalent of (1) is presented with the expression

$$u(k)=K_p\cdot[b\cdot r(k)-y(k)]+\frac{b_{i1}+b_{i2}\cdot q^{-1}}{1-q^{-1}}\cdot[r(k)-y(k)]+$$
$$+\frac{b_d\cdot\left(1-q^{-1}\right)}{1-a_d\cdot q^{-1}}\cdot[c\cdot r(k)-y(k)], \qquad (2)$$

where the coefficients b_{i1}, b_{i2}, a_d and b_d are formed from table 1, on condition that T_0 is the sample time for working out the digital PID [2]. According to three of the main discretization methods (see table 2) the general coefficients γ_i and γ_d have values 0, 0.5 or 1, but they can additionally be tuned.

The controller has a pole a_d and to be stable it is necessary the condition ½a_d½ ≤ 1 to be

Table 1. Formed of the coefficients b_{i1}, b_{i2}, a_d and b_d

$b_{i1}=K_p\cdot\frac{T_0}{T_i}\cdot\gamma_i$	$b_{i2}=K_p\cdot\frac{T_0}{T_i}\cdot(1-\gamma_i)$
$a_d=1-\frac{1}{\frac{T_f}{T_0}+\gamma_d}$	$b_d=K_p\cdot\frac{T_d}{T_0}\cdot\frac{1}{\frac{T_f}{T_0}+\gamma_d}$

Table 2. The main discretization methods

Coefficients	Forward difference (*Eurler–1*)	Backward difference (*Eurler–2*)	Bilinear transformation (*Tustin*)
γ_i	0	1	0.5
γ_d	0	1	0.5

fulfilled. It is proved [2] that the controller (2) may realize unstable control signal only, first, if the differentiation part of the digital PID is defined by forward difference approximation method, and second, if the inequality

$$T_f \geq \frac{T_0}{2} \tag{3}$$

is not fulfilled. On the basis of suggested in [1] proportion $T_f = T_d \,/\, N$ (N = 8 ¸ 20) from (3) can be drawn condition for choice of T_0

$$T_0 \leq \frac{2T_d}{N} = \frac{T_d}{4 \div 10}.$$

The universal digital PID can be worked out in the form of discrete difference equation

$$u(k) = p_1 \cdot u \cdot (k-1) + p_2 \cdot u \cdot (k-2) + t_0 \cdot r(k) + t_1 \cdot r(k-1) + \\ + t_2 \cdot r(k-2) + s_0 \cdot y(k) + s_1 \cdot y(k-1) + s_2 \cdot y(k-2) \tag{4a}$$

which coefficients are defined by the expressions 4b, [3].

The implementing of anti-windup mechanism in the universal digital PID controller is necessary to avoid the negative effect of the surplus increasing value of the integral part, when the

$$\begin{aligned} & p_1 = (1 + a_d); \\ & p_2 = -a_d\,; \\ & t_0 = K_p \cdot b + b_{i1} + b_d c; \\ & t_1 = -K_p \cdot b \cdot (1 + a_d) - b_{i1} \cdot a_d + b_{i2} - 2 \cdot b_d \cdot c; \\ & t_2 = K_p \cdot b \cdot a_d - b_{i2} \cdot a_d + b_d \cdot c; \\ & s_0 = -K_p - b_{i1} - b_d\,; \\ & s_1 = K_p \cdot (1 + a_d) + b_{i1} \cdot a_d - b_{i2} + 2 \cdot b_d\,; \\ & s_2 = -K_p \cdot a_d + b_{i2} \cdot a_d - b_d\,. \end{aligned} \tag{4b}$$

control signal comes out from the imposed in ACS bounds. The well-known [1] scheme solution for the analog PID is transferred for the digital one (see figure 1). The implemented negative loop from the control signal to the integrator causes the so called "anti-wind up effect" after physical limitations on the signals and it proves expected efficiency in the discrete case too [6].

3. Sample time in hybrid ACS

The discrete–analog or hybrid ACS presents a system for control of analog plant with discrete controller. The accuracy of the analog PID discretization will define its effectiveness on the implementation of the digital PID. In this chapter research works on the behavior of signals are made to show the difference due to the inappropriate choice of sample time in a hybrid ACS with digital PID and in an analog ACS with analog PID. The conclusions explain the following searching procedure of optimal sample time in chapter 4 and

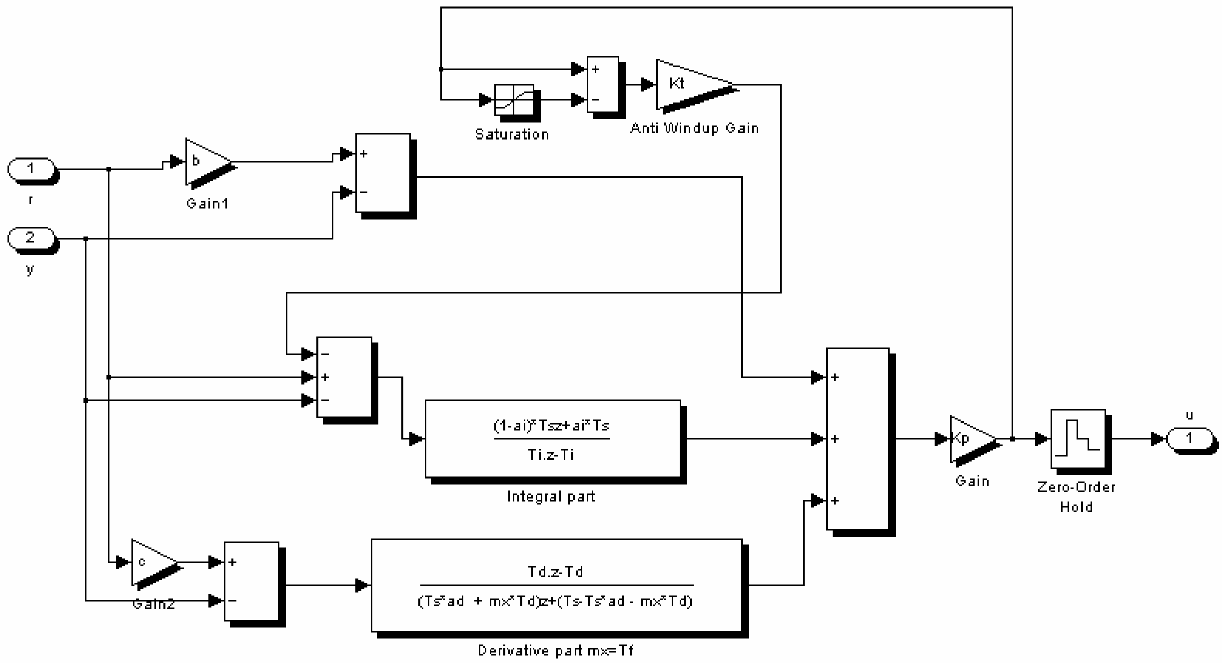


Fig. 1. Scheme of the universal digital PID controller with implemented anti–windup mechanism

lay in the base of two-rate discrete-analog ACS realization in chapter 5. Three plants with strongly different dynamics are used to perform comparison of the control and controlled signals in each analog ACS, on the one hand, with the corresponding signals in hybrid ACS with different sample times, on the other hand. Only few results will be presented but all of them are in [6].

3.1. ACS of a stable first order plant with time delay

Let the plant is presented by the transfer function

$$W(p) = \frac{1}{12p+1} e^{-2p}. \qquad (5)$$

The observed and presented in graphics signals (on the left – control signal, on the right – controlled signal) from analog ACS with appropriate tuned PID controller (table 4a) are compared with the corresponding ones from hybrid ACS with appropriate tuned digital PID controllers (table 4b). (After T_0 = 4 seconds the digital ACS becomes unstable).

Table 4a. The tuned ACS with analog PID controller

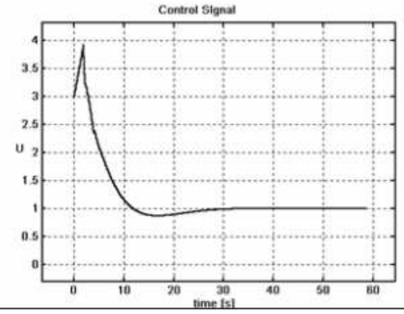
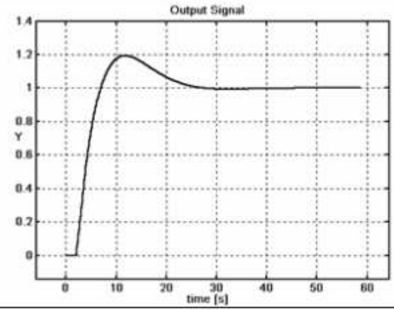
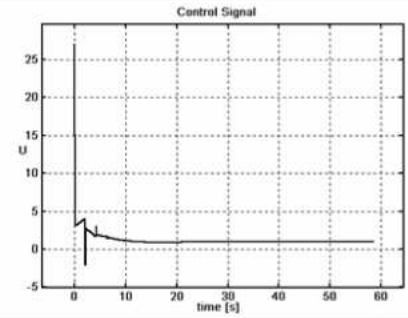
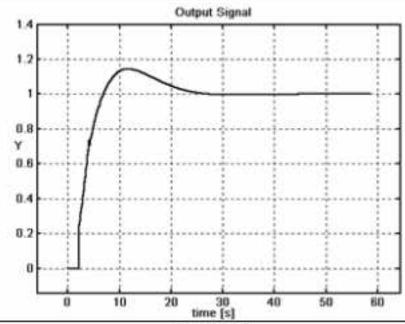
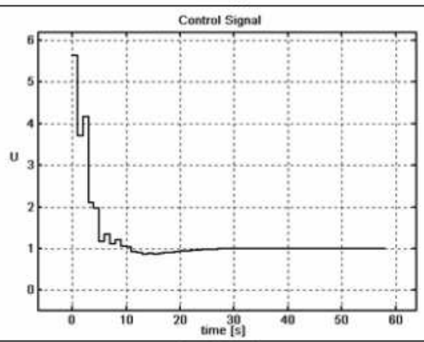
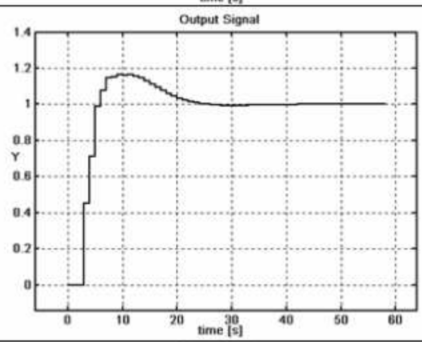
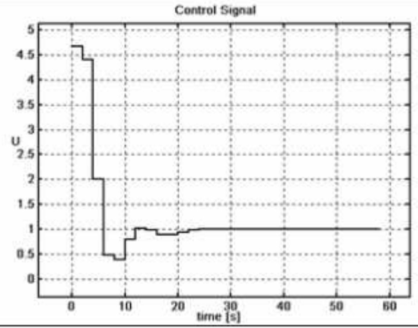
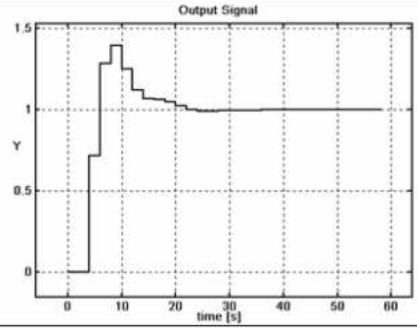
	

Table 4b. ACS with discrete PID

0.1		
1		
2		

3.2. ACS of a stable third order plant with oscillations

Let the plant is presented by the transfer function

$$W(p) = \frac{1}{4 \cdot p^3 + 1.38 \cdot p^2 + 4.095 \cdot p + 1}. \qquad (6)$$

The graphics by analogy of these in tables 4a and 4b are given in tables 5a and 5b. (After $T_0 = 2$ seconds the digital ACS becomes unstable).

3.3. ACS of an unstable second order plant

Let the object is presented by the transfer function

$$W(p) = \frac{1p + 1}{10p^2 - 5p + 1}. \qquad (7)$$

The graphics by analogy of these in tables 4a and 4b are given in tables 6a and 6b. (After $T_0 = 1$ second the digital ACS becomes unstable).

3.4. Conclusions

According the experiments one can see that the sample time of the PID controller has its optimal value for each plant. Changing up or down this value causes specific problems for the hybrid ACS. Thus, the decreasing sample time leads to the coincidence between the controlled signal behavior of the hybrid and the analog ACS. But the designed hybrid ACS could have inner instability because of the increasing extreme values in the control signal and its impossible physical realization. And contrary, the increasing sample time influences badly on the PID controller discretization, the hybrid ACS signals become too different to same analog ACS ones, so the using of discrete instead of analog PID controller looses it's sense. Something more, in case of relatively large sample time the hybrid ACS gets unstable due to the relatively large losses of the information. The experiments confirm the logical conclusion, that

Table 5a. The tuned ACS with analog PID controller

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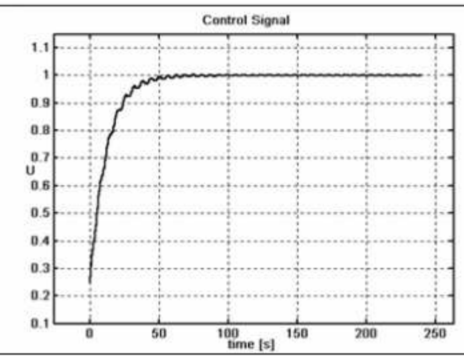
<td>
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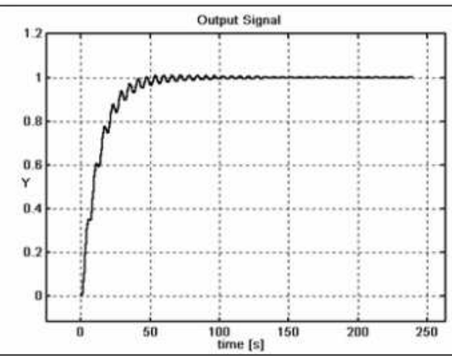
<td>
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Table 5b. ACS with discrete PID

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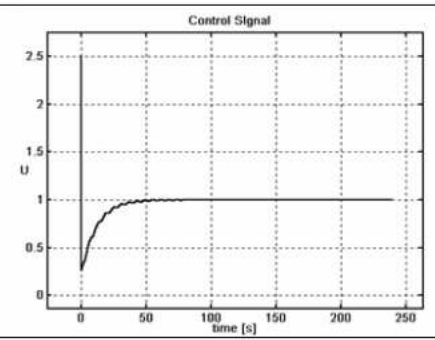
<td>
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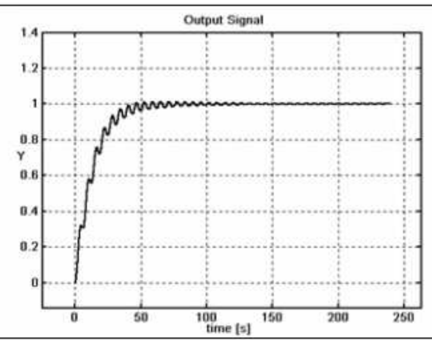
<td>
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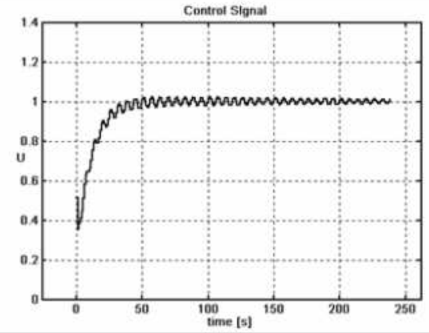
<td>
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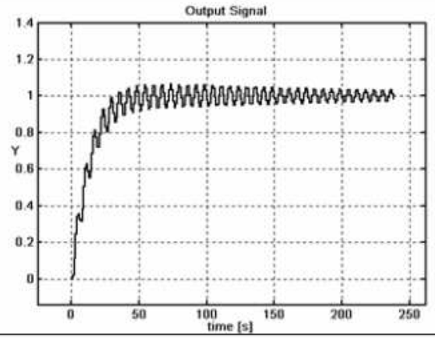
<td>
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Table 6a. The tuned ACS with analog PID controller

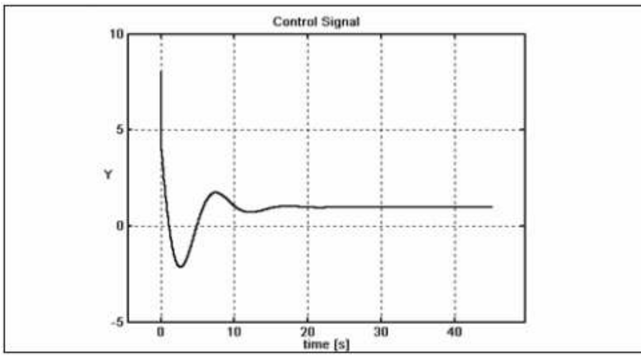
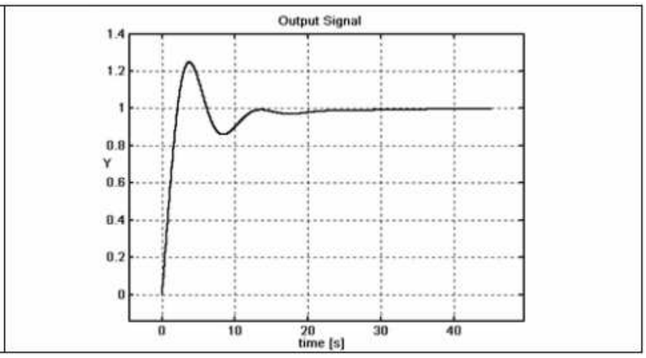
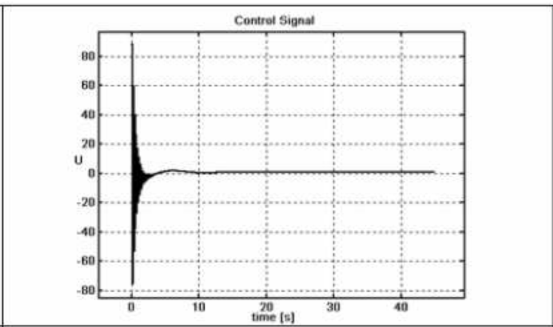
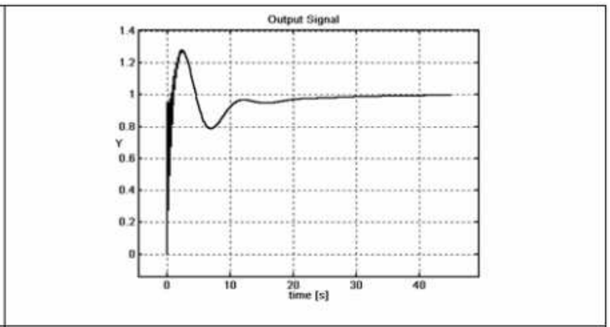
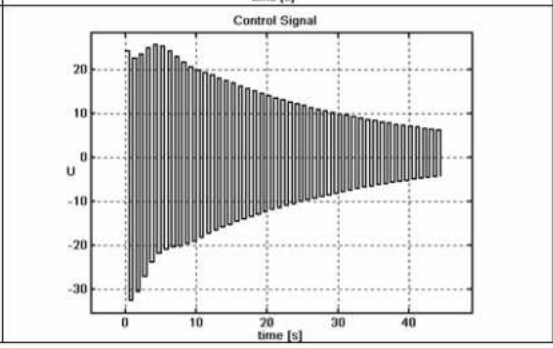
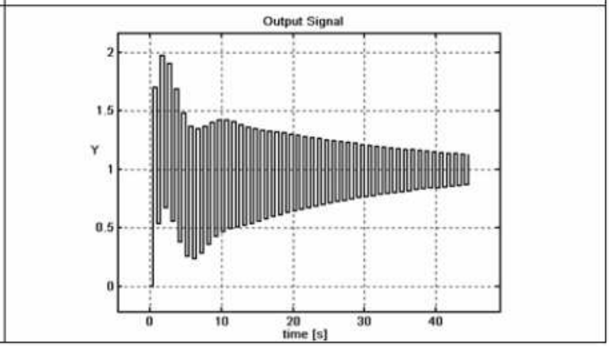
	

Table 6b. ACS with discrete PID

0.1		
0.5		

the influence of the sample time is more strongly expressed over ACS with smaller stability margin.

4. Choice of optimal sample time

The following task is defined: using the optimization procedure to find down that sample time which causes the best behavior of the hybrid ACS (with universal digital PID controller) comparing to the analog ACS (with universal analog PID controller) both in respect of the controlled and control signals. Because of this reason the following complex performance index of quality is suggested [5]:

$$J=\sqrt{\left(\int_0^\infty \left| y_A(t)-y_H(t)\right| dt\right)^2+\left(\alpha\int_0^\infty \left| u_A(t)-u_H(t)\right| dt\right)^2},\quad \alpha>0, \qquad (8)$$

with the outputs and the inputs of the plant in the analog (A) and hybrid (H) CSA, which are included into integrals of the corresponding absolute errors due only to the optimal sample time T_0 = variable. The weight coefficient α balances the influence of the different signals types over the performance index.

With choice of small sample time, this index has substantial offset – it doesn't succeed to punish enough the relatively large extremes values of the digital control signal at the first sample. The area bounded of the control signal at this moment does not increase quickly enough and the index (8) practically does not read the extremes values. That is why the modified quality performance index is suggested,

$$J^* = J + \max(|\, u_H(t)\, |), \qquad (9)$$

by adding an extra "area" in (8) with implementation of the maximum absolute value of the digital control signal.

Three different optimization methods are used with the standard functions ***fmincon*** and

fminsearch from Optimization Toolbox in the test examples. Each procedure has to find out the optimal sample time in the ACS starting from initial conditions chosen in the interval [T_{0L} ... T_{0U}]:

$$T_{0ini} = (T_{0L} + T_{0U})/2;$$
$$T_{0ini} = \sqrt{T_{0L} \cdot T_{0U}}; \qquad (10)$$
$$T_{0ini} = T_{0L}; \qquad T_{0ini} = T_{0U}.$$

4.1. ACS of a stable first order plant with time delay

The index of quality J (8) and J^* (9) shown in figure 2 are minimized in respect of the sample time T_0 = var = T_s.

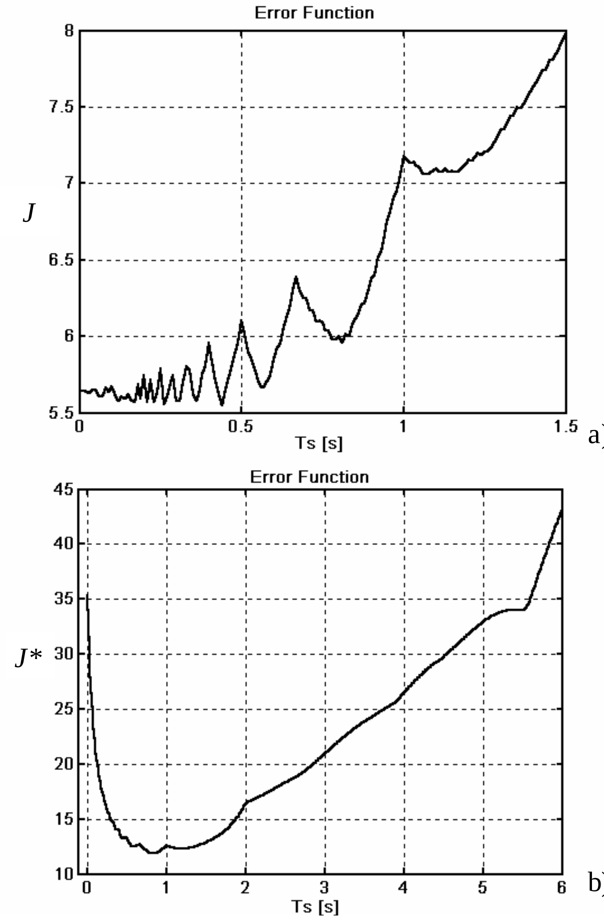


Fig. 2. Performance index

The index J depends on the plant dynamics and all additional parameters of the digital PID controller. It has so many local minima and this presents real difficulty for optimization. And vise versa the modified index J^* is relatively smooth unimodal function so ***fmincon*** and ***fminsearch*** deal with the task without problems. The optimal sample time as a solution choice is given in table 7 and the ACS signals are drawn in figure 3.

Table 7. The optimal sample time as a solution choice

	$T_{0\,opt}$	J^*
fmincon	0.834627	11.829320
fminsearch	0.834660	11.829391

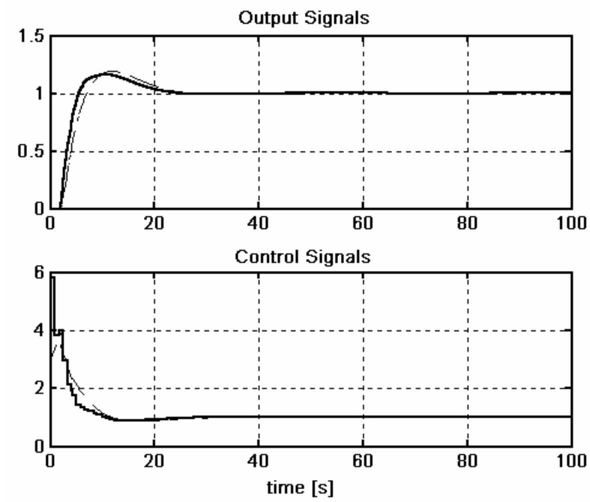


Fig. 3. y and u of the analog (the dotted line) and the optimized ACS

4.2. ACS of a stable third order plant with oscillations

For this plant the index of quality J (8) and J^* (9) in respect of the sample time T_0 = var = T_s have the form shown in figure 4. One can see that in case (a) this optimization function has a local minimum, in which there is real danger to "lodge", if the initial value of the sample time $T_{0\,ini}$ has not chosen quite good. Thus, for example, the ***fminsearch*** finds only the local minimum, if $T_{0\,ini} \subset$ [1.5 … 2.5]. For bigger values the error is large enough to accelerate the searching procedure, which jumps over the fatal local minimum and the procedure succeeds finding out the true solution as it manages to do it for smaller initial values. But the global minimum is related with the smallest possible sample time, which is a disadvantage for the hybrid ACS because of the control signal high level at the initial operating moment following the reference. The range of searching for the optimal T_0 can be limited by ***fmincon*** so in case (b) the appearance of the index is improved. The global minimum is shifted a little bit in right direction to the bigger sample times and the local minimum becomes "safer" for the searching procedure. Results are shown in table 8 and figure 5.

4.3. ACS of a unstable second order plant

For this plant the index of quality J (8) and J^* (9) in respect of the sample time T_0 = var = T_s

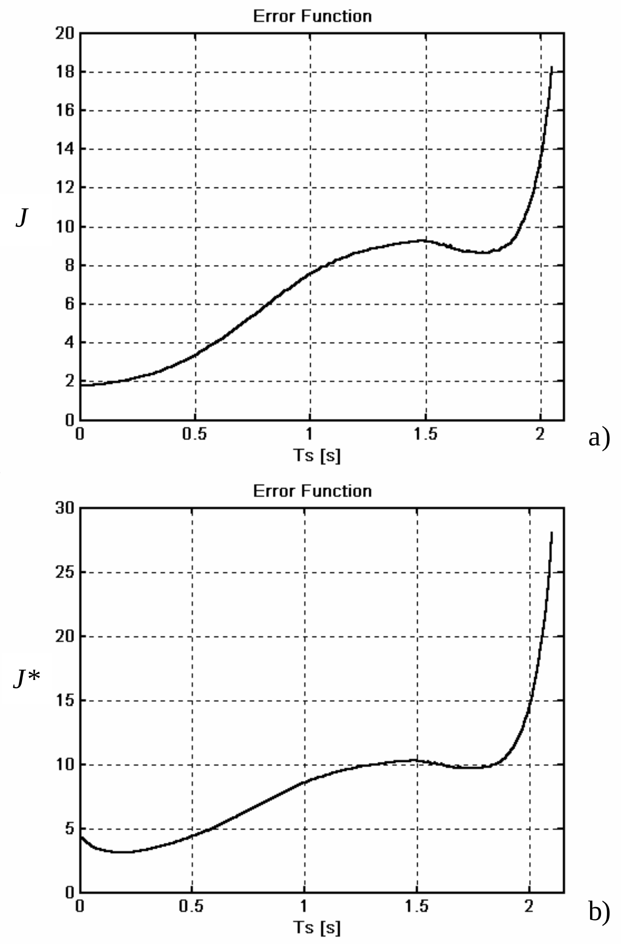


Fig. 4. Performance index

Table 8. The optimal sample time as a solution choice

	$T_{0\ opt}$	J^*
fmincon	0.205257	3.091925
fminsearch	0.205841	3.092903

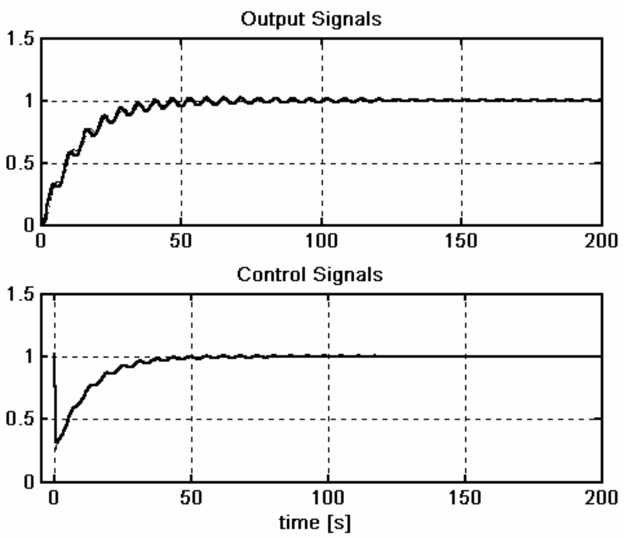


Fig. 5. *y* and *u* of the analog (the dotted line) and the optimized ACS

have the form shown in figure 6. One can see that in case (a) this optimization function is easy for finding out the global minimum, but the small optimal value of the sample time is unacceptable because of the high control signal amplitudes. In case (b) the index represents a unimodal function with a minimum within the capacity of any optimization procedure. Results are shown in table 9 and figure 7.

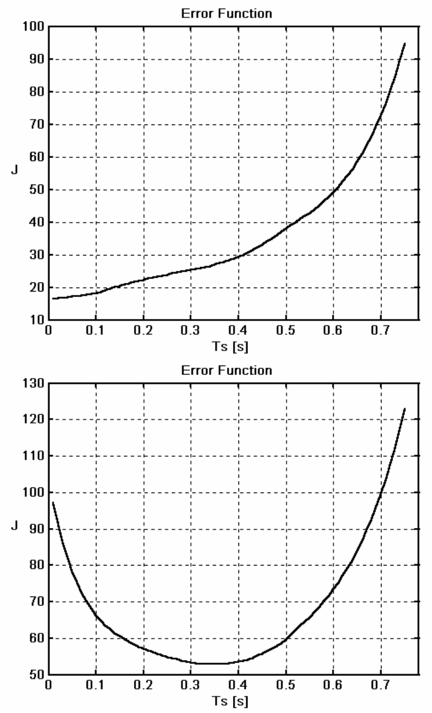


Fig. 6. Performance index

Table 9. The optimal sample time as a solution choice

	$T_{0\ opt}$	J^*
fmincon	0.340828	52.839472
fminsearch	0.340924	52.839605

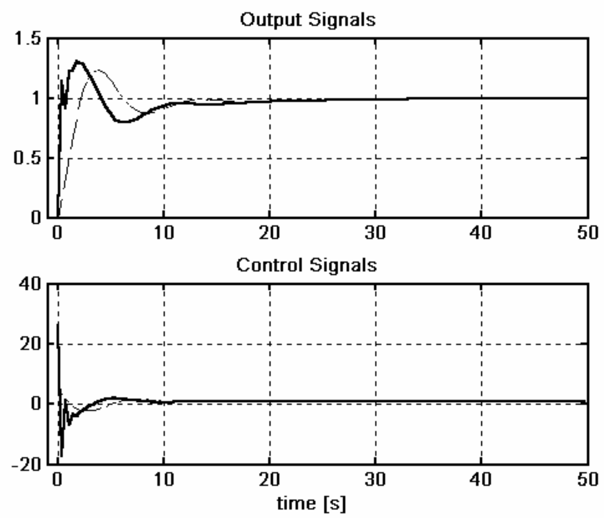


Fig. 7. The output and control signals of the analog (the dashed line) and the optimized ACS

Table 10. Signals in one-rate hybrid ACS

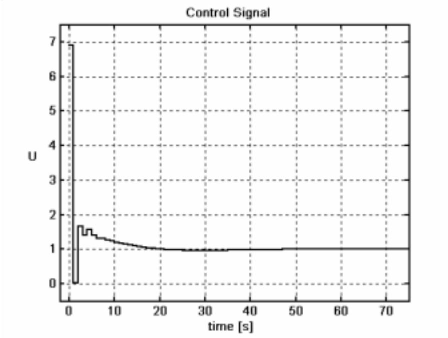
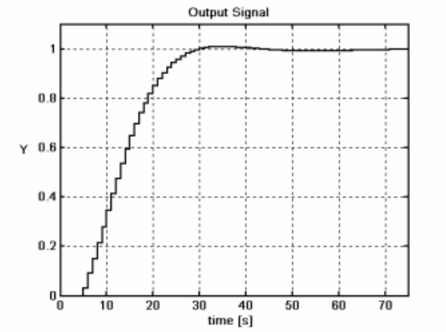
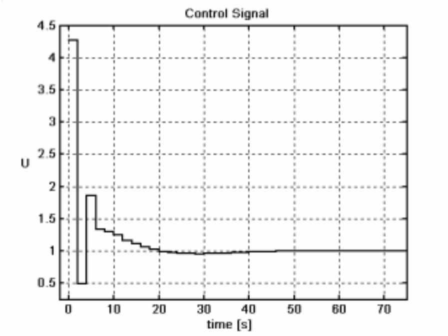
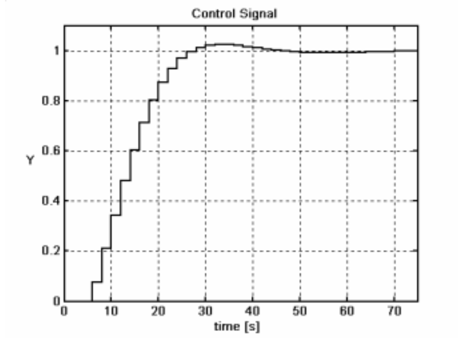
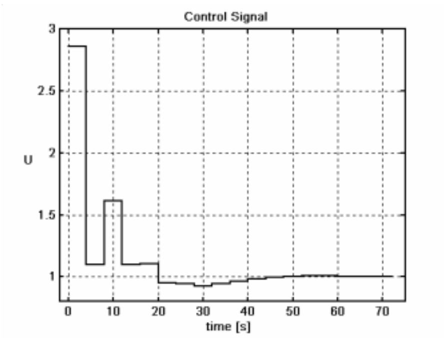
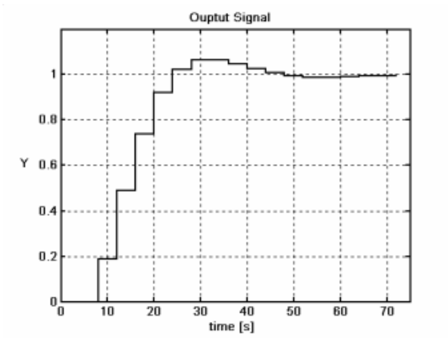
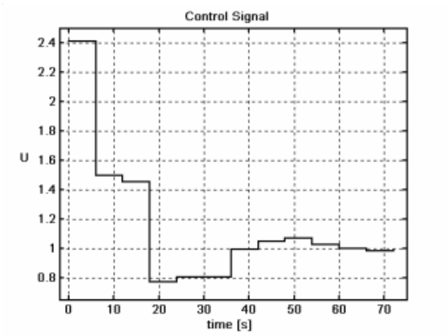
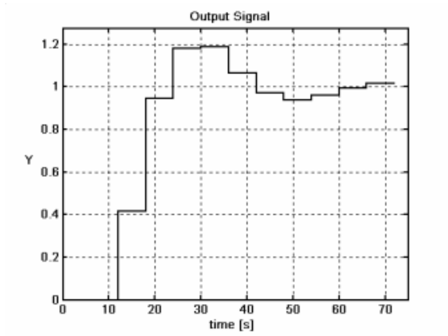
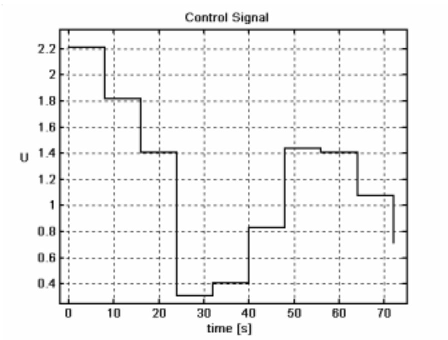
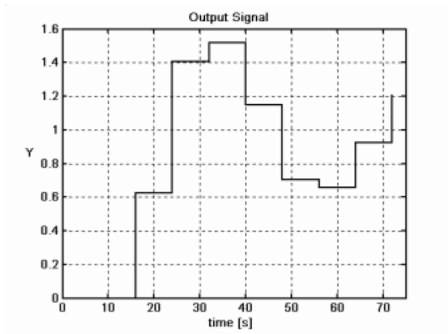
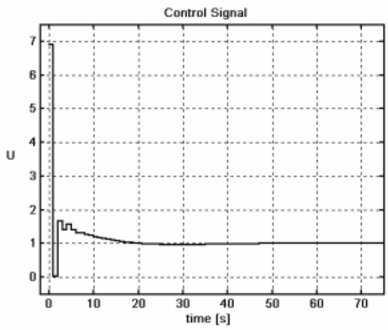
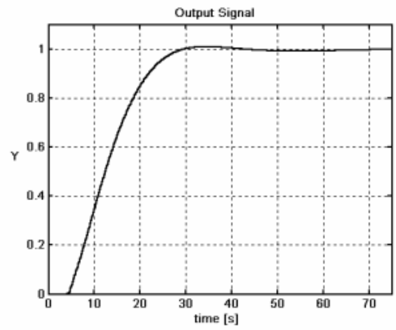
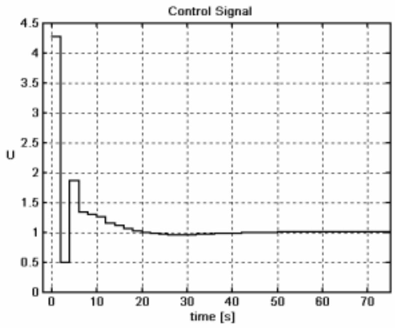
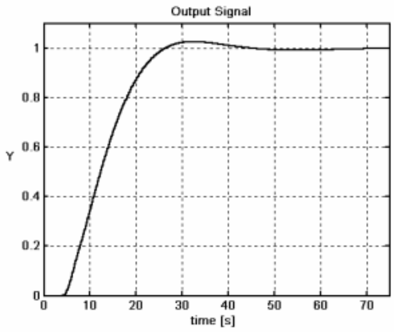
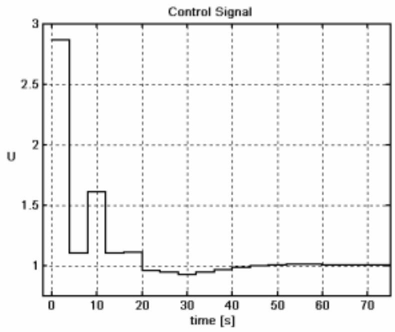
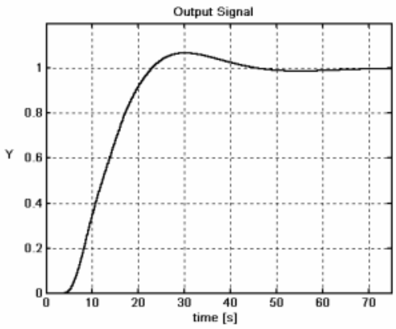
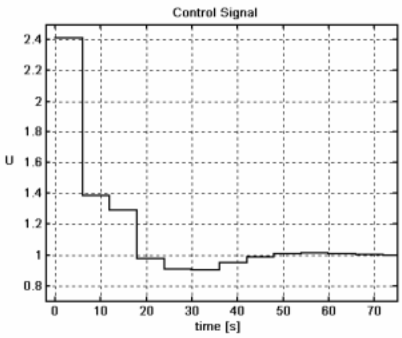
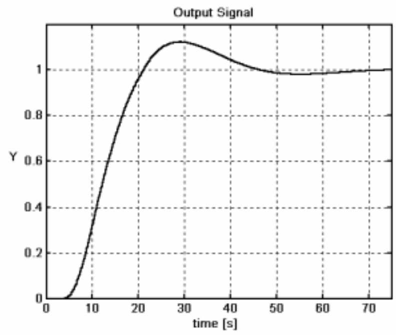
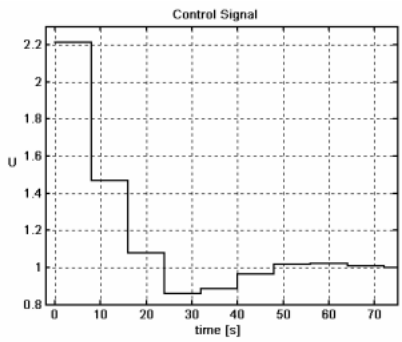
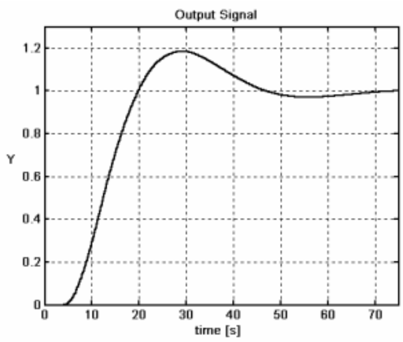
1		
2		
4		
6		
8		

Table 11. Signals in two–rate hybrid ACS

1		
2		
4		
6		
8		

4.4. Conclusions

It is obvious that an optimal T_0, in which the output signal in a system with digital PID is very close to the same one in a system with analog PID, always exists but to find it is up to the optimization procedure. The sample time depends as on the performance index as on the tuning parameters of the digital PID controller. To calculate the correct value of T_0 depends also on the optimization method (function) and the initial conditions in it as well.

Sometimes it is preferable to go to the intermediate suboptimal results which give balance acceptable behavior of the hybrid ACS.

5. Two–rates ACS

In the real technological processes control by industrial controllers the main sample time of functioning of ACS is of the order of 100 ms. For hybrid ACS of dynamical plant this leads to control signals to be formed with relatively large amplitudes when the reference signal has step changes. One good solution of the problem is the controller to work with a sample time multiple times bigger than the small sample time of the whole ACS. Such a control system is called two-rate in contrast to the classical one-rate ACS.

The described above idea is implemented on a plant with the following transfer function

$$W(p) = \frac{K \cdot (T_4 \cdot p + 1)}{(T_1 \cdot p + 1) \cdot (T_2 \cdot p + 1) \cdot (T_3 \cdot p + 1)} \cdot e^{-\tau p} \quad (11)$$

$K = 1$; $T_1 = 10$ s; $T_2 = 7$ s; $T_3 = 3$ s; $T_4 = 2$ s; $\tau = 4$ s.

The digital PID controller (2) is tuned with parameters: $K_p = 1.386$, $T_i = 15.928$; $T_d = 3.353$; $b = 0.848$; $c = 1$; $T_i = 0.1$. Its sample time T_0^{PID} could match (one-rate ACS) or be multiple times bigger (two-rate ACS) the sample time T_0^{ACS} of the ACS.

5.1. One–rate ACS

In table 10 are given the signals (left – control signal, right – controlled signal) of one-rate hybrid ACS of the test plant in cases of different sample times. (After $T_0^{PID} = T_0^{ACS} = T_0 = 6$ seconds ACS begins functioning badly).

5.2. Two–rates ACS

In table 11 are given the signals (left – control signal, right – controlled signal) of two-rate ACS of the test plant in cases of different sample times. T_0^{PID} = var., $T_0^{ACS} = 0.1$ s.

5.3. Conclusions

The test two–rate ACS shows week sensitiveness to the increasing sample time of the controller, in contrast of the one–rate. Through two–rate ACS the system output gives smoother response to the reference signal, without getting extremes values of the control signal.

6. Summary

On simulation examples the tests are made to control the different dynamical analog plants by analog or digital PID controllers. The emphasis is laid on the definition of an optimal sample time for the hybrid ACS. The suggested two–rate ACS shows improved behavior in comparison with the one–rate.

References

1. Astrom, K., Hagglund, T.: *PID Controllers*. Second Edition. Instrument Society of America, 1995
2. Garipov, E., Tomov, L.: *Design and Implementation of a Universal Digital PID Controller Formed by Generalized Discretization*. Proc. of the Technical University Sofia, Bulgaria, Vol. 54, no. 1, 2004, p. 25-32 (in Bulgarian)
3. Garipov, E.: *Digital Control Systems*. Technical University Sofia, Bulgaria, 2004 (in Bulgarian)
4. Izermann, R.: *Digital Control Systems*. Mir, USSR, 1984 (in Russian)
5. Tomov, L.: *Program System for Analogue/Digital PID Controllers Design*. BEng Thesis. Technical University Sofia, 2003 (in Bulgarian)
6. Tomov, L.: *Research on the Universal Digital PID Controllers*. MEng Thesis. Technical University Sofia, Bulgaria, 2005 (in Bulgarian)

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