ASPECTS REGARDING THE DECISION ON INSURING AUTO FLEETS BELONGING TO COMMERCIAL COMPANIES

Florin-Cătălin OLTEANU, Gavrilă CALEFARIU, Flavius Aurelian SÂRBU
Transilvania University of Brasov, Romania

Abstract. The decisions that an insurance company has to make regarding the insurance of auto fleets belonging to various companies are decisions taken in conditions of uncertainty and risk. This paper presents the use of Bayes decision procedures in the case of such choices. These procedures include the Bayes decision procedure with no experimentation and initial probabilities as well as the Bayes decision procedures with experimentation and conditioned and post probabilities. The paper also presents a numerical application in which it is shown how an insurance company can evaluate, using these procedures, an insurance request against theft and damage for an auto fleet of an industrial production company.

Keywords: theft and damage insurance, decision, uncertainty, risk, Bayes procedure, initial probabilities, conditioned probabilities, post probabilities

1. Introduction

The insurance against theft and damage (CASCO) is important to companies both because of the inherent risk of auto accidents as well as for the high costs incurred by fixing the damaged vehicles.

This insurance covers the following risks [1]:

- accidents: collisions with other vehicles or other mobile or immobile bodies;
- fire, lightning, explosion;
- theft;
- natural disasters: torrential rain, hailstorm, flood, storm, earthquake, landslide, etc...

On the other hand, the insurance companies which do cover vehicles are confronted with a slew of problems generated by a series of factors, such as: the high frequency of collisions, the high costs related to damages, the irresponsibility of certain drivers, the vehicles' technical state, and the road infrastructure's state [2]. These factors make it so that the decision to accept a request from an industrial production company or from any other commercial entity for their auto fleet presents a certain risk factor for the insurer. At present, the decisions on closing such insurance policies are taken based on risk inspections and on the auto fleet's damage record analysis.

The damage rate shows the ratio between the damages paid by the insurer and the collected premiums [3], and it is calculated with the relation:

$$R_D = \frac{D}{P} \times 100, \qquad (1)$$

where: D - all the damages paid by the insurer, and P - all the premiums cashed by the insurer.

This paper aims to present a few aspects related to the problem of the decision on insuring commercial companies' auto fleets by using the Bayes decision procedures. These procedures transform the decisions taken in uncertainty conditions to decisions with risk conditions by using initial probability distribution, resulted from the insurer's own experience, as well as by using conditioned probabilities and post distribution, resulted from statistical experiments regarding the decision problem.

The specific algorithm used in this paper to solve the decision problem in the case of insurance is based on a general algorithm that's presented in detail in the cited literature [4, 5, 6].

This methodology lends itself to computer use. Thus a software system which can be useful to insurance companies has been created.

2. Formulating the decision problem

Formulating the decision problem requires: establishing the action space, the state space, the consequences and the loss function.

The action space, $\mathcal{A} = \{a_1, a_2\}$, includes the following actions:

a_1 - the insurer closes the policy;
a_2 - the insurer does not close the policy.

The state space, $\boldsymbol{\Theta} = \{\theta_1, \theta_2, \theta_3\}$ can have the following states according to the potential damages rate of the insured fleet during the contract's duration:

θ_1 - there are no damages;
θ_2 - there are damages and the damage rate is $R_D < 100\%$;
θ_3 - there are damages and the damage rate is $R_D > 100\%$.

Notation: P - total premium for the entire fleet; R_{Di} - damage rate for each state θ_i, where i=1,2,3; C_0 - the costs of risk inspections and offer arrangements; P_i - profits for each state. In the case of action a_1 these profits are:

$$P_i = P \cdot (1 - R_{Di}). \qquad (2)$$

The consequences (θ_i, a_j) are defined, where j = 1,2, and the insurer's costs and profits associated to these consequences are shown in Table 1.

Table 1. Profits and costs

Actions	Nature's states		
	θ_1	θ_2	θ_3
a_1	P_1	P_1	P_1
a_2	C_0	C_0	C_0

The loss function is established, which is presented in Table 2 and in which the following notations are used: $L(\theta_1, a_1) = -P_1$; $L(\theta_2, a_1) = -P_2$; $L(\theta_3, a_1) = -P_3$; $L(\theta_1, a_2) = L(\theta_2, a_2) = L(\theta_3, a_2) = -C_0$.

Table 2. The loss function

Actions	Nature's states		
	θ_1	θ_2	θ_3
a_1	$L(\theta_1, a_1)$	$L(\theta_2, a_1)$	$L(\theta_3, a_1)$
a_2	$L(\theta_1, a_2)$	$L(\theta_2, a_2)$	$L(\theta_3, a_2)$

3. The decision with no experimentation

In the case of the decision without experimentation the Bayes decision procedure with initial probabilities is applied.

The decider may have information on the manifestation possibilities of nature's states θ_i, which they could transform into a probability distribution of their conviction regarding the said possibilities called initial distribution and noted Prob{$\theta=\theta_i$} = $p_\theta(i)$ [4]. This distribution is often subjective and depends on the deciders' experience and intuition. The initial distribution, seen in Table 3, must meet these conditions $0 \le p_\theta(i) \le 1$ and $\sum_i p_\theta(i) = 1$.

Table 3. Initial probabilities

Nature's state	θ_1	θ_2	θ_3
Initial probabilities	$p_\theta(1)$	$p_\theta(2)$	$p_\theta(3)$

The medium loss for each action a_j is calculated with (3):

$$L(a_j) = M[L(\theta_i, a_j)] = \sum_{i=1}^{3} L(\theta_i, a_j) \cdot p_\theta(i). \qquad (3)$$

in the last column of Table 4. The minimum medium loss min$L(a_j)$ is determined and, according to the procedure, the option with the least losses is chosen.

Table 4. Determining the medium loss

	Nature's states			$L(a_j)$
	θ_1	…	θ_3	
a_1	$L(\theta_1, a_1)p_\theta(1)$	...	$L(\theta_3, a_1)p_\theta(3)$	$L(a_1)$
a_2	$L(\theta_1, a_2)p_\theta(1)$	...	$L(\theta_3, a_2)p_\theta(3)$	$L(a_2)$

4. The decision with experimentation

The sampling space $\mathcal{X}$, and the random variable X is defined, where X = 1, 2, 3, ... k, ... In the insurance companies' experience, the drivers' age is a high risk factor which can lead to accidents, younger drivers being the most exposed. Insurance companies take this factor into account when the premiums for the mandatory policies (RCA) are established. They are age differentiated, younger drivers having to pay more than older ones for the same vehicle type.

Taking into account the medium age of the drivers, the companies' fleets can be placed in one of the following categories:

X = 1 – low risk fleet, drivers' medium age over 35;
X = 2 – medium risk fleet, drivers' medium age between 25 and 35;
X = 3 – high risk fleet, drivers' medium age under 25.

Table 5 shows the conditioned probabilities resulted from studies or from the insurance companies' experience regarding the drivers' age influence on the possibility of accidents.

Table 5. Conditioned probabilities

X	θ_1	θ_2	θ_3
1	$\pi_{\theta_1 \mid X=1}$	$\pi_{\theta_2 \mid X=1}$	$\pi_{\theta_3 \mid X=1}$
2	$\pi_{\theta_1 \mid X=2}$	$\pi_{\theta_2 \mid X=2}$	$\pi_{\theta_3 \mid X=2}$
3	$\pi_{\theta_1 \mid X=3}$	$\pi_{\theta_2 \mid X=3}$	$\pi_{\theta_3 \mid X=3}$

The manifestation probability of state θ_i is noted $\pi_{\theta_i \mid X=k}$ - if the fleet's group, according to its drivers' age, is $X = k$.

4.1. The Bayes decision procedure with conditioned probabilities

For each action a_j, the medium risk function is determined with (4) in Table 6 for each evaluation $X = k$, k = 1,2,3 [7].

$$R(a_j \mid X = k) = \sum_{i=1}^{n} L(\theta_i, a_j) \cdot \pi_{\theta_i \mid X=k}. \qquad (4)$$

In order to establish the decision rule $d_n(x)$, the minimum values of the medium risk from each column in Table 6 is determined.

Table 6. Medium risk

	$X = 1$	$X = 2$	$X = 3$
a_1	$R(a_1 \| X=1)$	$R(a_1 \| X=2)$	$R(a_1 \| X=3)$
a_2	$R(a_2 \| X=1)$	$R(a_2 \| X=2)$	$R(a_2 \| X=3)$

The actions, $a_{jk} \in \mathcal{A} = \{a_1, a_2\}$, corresponding to these minimum values are chosen and, according to procedure, the decision rule presented in Table 7 is established.

Table 7. The decision rule

X	Chosen action	Minimum medium risk
1	a_{j1}	$\min_k\{R(a_j \| X=1)\}$
2	a_{j2}	$\min_k\{R(a_j\{X=2)\}$
3	a_{j3}	$\min_k\{R(a_j \| X=3)\}$

4.2. The Bayes decision procedure with post probabilities

For this procedure both the conditioned post probabilities, $\pi_{\theta_i|X=k} \times p_\theta(i)$, as well as the marginal distribution $\pi_X(k)$, are calculated with relation (5):

$$\pi_X(k) = \sum_{i=1}^{n} (\pi_{\theta_i|X=k} \times p_\theta(i)). \quad (5)$$

The post distribution is calculated with (6):

$$f_{\theta|X=k}(i) = \frac{\pi_{\theta|X=k} \times p_\theta(i)}{\pi_X(k)}, \quad (6)$$

and the data is organized in Table 8.

Table 8. Post distribution

	$f_{\theta\|X=k}(i)$		
X	θ_1	θ_2	θ_3
1	$f_{\theta\|X=1}(1)$	$f_{\theta\|X=1}(2)$	$f_{\theta\|X=1}(3)$
2	$f_{\theta\|X=2}(1)$	$f_{\theta\|X=2}(2)$	$f_{\theta\|X=2}(3)$
3	$f_{\theta\|X=3}(1)$	$f_{\theta\|X=3}(2)$	$f_{\theta\|X=3}(3)$

The Bayes risk is determined, according to the post distribution, for each action a_j, in the case of each evaluation $X = k$, with (7):

$$B_f(a_j | X = k) = \sum_{i=1}^{n} L(a_j, \theta_i) f_{\theta_i|X=k}(i), \quad (7)$$

and the data are organized in Table 9.

Table 9. The Bayes risk

	$X = 1$	$X = 2$	$X = 3$
a_1	$B_f(a_1 \| X=1)$	$B_f(a_1 \| X=2)$	$B_f(a_1 \| X=3)$
a_2	$B_f(a_2 \| X=1)$	$B_f(a_2 \| X=2)$	$B_f(a_2 \| X=3)$

The decision rule is established by choosing for each value of X the action $a_{jk} \in \mathcal{A} = \{a_1, a_2\}$, with the minimum Bayes risk. The decision rule, according to the Bayes procedure, is shown in Table 10.

Table 10. The decision rule

X	Chosen action	Bayes risk
1	a_{j1}	$\min_k\{B_f(a_j \| X=1)\}$
2	a_{j2}	$\min_k\{B_f(a_j\{X=2)\}$
3	a_{j3}	$\min_k\{B_f(a_j \| X=3)\}$

5. Numeric application

The insurance company „ALFAASIG” receives a request for offer from the „GAMA” company for the insurance of the latter's auto fleet, composed of 25 vehicles, against damage and theft (CASCO). The total value of the premiums resulted from calculating the offer is P = 30,000 euro. The damage rates for the three states are R_{D1} = 10%, R_{D2} = 60% şi R_{D3} = 200%.The costs for the risk inspection and for arranging the offer is C_0 = 200 euro. The profits, calculated with (2), and costs of the insurance company are shown in Table 11.

The loss function is presented in Table 12.

Table 11. Profits and costs

	θ_1	θ_2	θ_3
a_1	27,000	12,000	– 30,000
a_2	-200	– 200	– 200

Table 12. The loss function

	θ_1	θ_2	θ_3
a_1	– 27,000	– 12,000	30,000
a_2	200	200	200

5.1. The decision with no experimentation

The Bayes decision procedure with initial probabilities is applied.

The initial probabilities $p_\theta(i)$, are presented in Table 13.

Medium loss is calculated for each action a_j, with relation (3), in the last column of Table 14.

Thus $\min L(a_j) = -13{,}500$. The action a_1 is chosen.

Table 13. The initial probabilities

Nature's state	θ_1	θ_2	θ_3
Initial probabilities	0.5	0.31	0.19

Table 14. Determining medium loss

Action	Nature's state			$L(a_j)$
	θ_1	θ_2	θ_3	
a_1	–13,500	–3,720	5,700	–11,520
a_2	100	62	38	200

5.2. The decision with experimentation

The conditioned probabilities resulted from the study $\pi_{\theta_i|X=i}$, i=1,2,3, are the ones in Table 15. A value from this table can be interpreted as follows: if X = 1, that is the drivers' medium age is higher than 35, the conditioned probability for the manifestation of state θ_2, meaning that damages would occur and the damage rate would be R_{D2} = 60%, is 7/40.

Table 15. Conditioned probabilities

X	θ_1	θ_2	θ_3
1	$\frac{31}{40}$	$\frac{7}{40}$	$\frac{2}{40}$
2	$\frac{10}{30}$	$\frac{14}{30}$	$\frac{6}{30}$
3	$\frac{5}{25}$	$\frac{8}{25}$	$\frac{14}{25}$

5.2.1. The Bayes decision procedure with conditioned probabilities

The medium risk for each action a_j, is determined with relation (4), for each evaluation $X = k$, in Table (16).

Table 16. Determining the medium risk

	X = 1	X = 2	X = 3
$R(a_1)$	–21,525	–8,595	9,720
$R(a_2)$	200	200	200

The adopted decision rule is shown in Table 17.

Table 17. The decision rule

X	Chosen action	Minimum medium risk
1	a_1	–21,525
2	a_1	–8,595
3	a_1	200

The significance of this decision rule is:
- if X = 1, a_1 is chosen;
- if X = 2, a_1 is chosen;
- if X = 3, a_2 is chosen.

5.2.2. The Bayes procedure with post probabilities

The post distribution is calculated with relation (6). Te result is shown in Table 18.

For each action, for each evaluation, the Bayes risk is calculated with relation (7). The results are shown in Table 19.

The minimum Bayes risk is determined and, in the case of each evaluation, the decision rule is established. The results are shown in Table 20.

Table 18. The probabilities' post distribution

Risk classification	$f_{\theta\|X=k}(i)$		
	θ_1	θ_2	θ_3
X = 1	0,859	0,120	0,021
X = 2	0,477	0,414	0,109
X = 3	0,226	0,373	0,401

Table 19. Determining the Bayes risk

	X = 1	X = 2	X = 3
$B_f(a_1)$	–23,997	–14,581	1,437
$B_f(a_2)$	200	200	200

Table 20. Establishing the decision

X	Bayes action	Bayes risk
1	a_1	-23,997
2	a_1	-14,581
3	a_2	200

The significance of the decision rule is:
- if X = 1, a_1 is chosen;
- if X = 2, a_1 is chosen;
- if X = 3, a_2 is chosen.

6. Conclusions

The usage of the damage rate indicator is useful in the decision process as it helps in the evaluation of auto fleets of different sizes.

The decision methodology used in this paper is complex and it makes use of the decider's experience and the information resulted from statistical experiments. Both initial and conditioned probabilities can be taken from studies, but it would be better if they were sourced from the insurance company's own statistics. In the latter case, the statistics on the damages rates must be kept in the zonal branches of the insurer as they can differ from region to region due to various factors, such as: the roads and their state, the degree traffic systematization, climatic factors, and others.

The use of all the procedures presented here or only of some of them to help solve the decision problem when analyzing insurance requests for auto fleets is in keep with the decider's attitude towards risk, in our case the insurer, as well as with the statistics owned by insurance companies regarding the damages rates for auto fleets.

The methodology used does not exclude decision procedures based on risk inspections, it complements them.

Acknowledgements

This paper is supported by the Sectorial Operational Program Human Resources Development (SOP HRD), financed from the European Social Fund and by the Romanian Government under the contract number POSDRU/88/1.5/S/59321.

References

1. Lungu, N.C. (2009) *Asigurări de bunuri* (*Insurance of goods*). ,,Alexandru Ioan Cuza" University Publishing House, ISBN 978-973-703-485-4, Iaşi, Romania (in Romanian)
2. Negoiță, I. (2001) *Aplicații practice în asigurări şi reasigurări* (*Practical applications in insurance and reinsurance*). Etape Publishing House, ISBN 973-9090-90-7, Sibiu, Romania (in Romanian)
3. Văcărel, I., Bercea, F. (1998) *Asigurări si reasigurări* (*Insurance and reinsurance*). Expert Publishing House, Bucureşti, Romania (in Romanian)
4. Bârsan-Pipu, N., Popescu, I. (2003) *Managementul riscului: concepte, metode, aplicații* (*Risk Management: Concepts, Methods, Applications*). Transilvania University Publishing House, ISBN 973-635-180-7, Braşov, Romania (in Romanian)
5. Bowker, A.H., Lieberman, G.J. (1972) *Engineering Statistics*. Prentice-Hall, ISBN: 978-0132794558
6. Hillier, F.S., Lieberman, G.J. (2004) *Introduction to Operations Research*. McGraw-Hill, 8 edition, ISBN: 978-0073017792
7. Olteanu, F.C., Calefariu, G., Fota, Adriana, Bârsan-Pipu, N. (2012) *Aspects regarding the decisional processin the insurance of industrial businesses*. Proceedings of the 16th International Conference Modern Technologies, Quality and Innovation Mod Tech 2012, ISSN 2069-6736, p. 697-700, Sinaia, România

Received in November 2012