The Using of the Chi Square Test to Verify the Normal Distribution of Some Experimental Results

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Abstract

The data presented in the paper are part of an ample study in regards to the susceptibility of fragile tear of a S.G. cast iron alloy with Ni-Cu-Cr, heat treated. The experimental data obtained from the experiments were analyzed with the help of the Chi-Square test for adjusting, in this case the analytic testing for the concordance of the distribution of the analyzed values with the normal distribution model. The specific steps for the calculus of the Chi-Square tests are presented and in the final phase by comparing the calculated value, χ^2_{calc} (8.0154) with the critical value, χ^2_{crit} (14.067) of the Chi-Square test, it follows that the condition: $\chi^2_{calc} < \chi^2_{crit}$ is satisfied, therefore assuming that the distribution of the experimental data studied follows a normal distribution.

1. Introduction

Verifying the character of the experimental distribution of random variables in a process can be studied by means of several statistical tests: Chi-Square, Kolmogorov-Smirnov, Massey [1]. One of the most used tests is the Chi-Square test, which can be applied in several situations, one of which being the comparison of an observed (or empirical) distribution on a theoretical distribution sample [1-11]. In this case the Chi-Square adjustment test is used, in which case the analytical testing of the concordance of the distribution of the analyzed values with the normal distribution pattern (Gauss) is attempted.

This test can also be used to compare two observed distributions to determine either the independence between two criteria or homogeneity in a contingency table, in which case the Chi-Square homogeneity or independence test is used.

In the present paper, the Chi-Square test will be used to verify that the experimental distribution assignment falls within a certain class of theoretical distribution (the most commonly used being the normal distribution).

2. Research Objectives

This main objective of this paper is to verifying the hypothesis regarding the normal distribution of analyze experimental data using the Chi-Square test. The analyzed data, are the specific results of the impact strength (KCU2) of a S.G. cast iron, heat treated [12, 13].

The test can be applied for samples of at least 50 values. Solving the test is done in the following steps [1, 11]:

(1) Sorting experimental data in ascending order with the determination of minimum (x_{min}) and maximum (x_{max}) and grouping of statistical data in classes or grouping intervals. The number of groups (n_g) used in this test is between 10 and 25 groups ($10 \leq n_g \leq 25$) and can be determined by H.A. Sturgers:

$$n_g = 1 + 3.222 \cdot \lg n , \qquad (1)$$

where n is the number of experimental determinations;

(2) Each group corresponds to a particular grouping interval (h) which is calculated with the relation:

$$h = \frac{x_{max} - x_{min}}{1 + 3.222 \cdot \lg n} = \frac{x_{max} - x_{min}}{n_g} , \qquad (2)$$

in which x_{max}, respectively x_{min}, is the maximum value, respectively the minimum value to each of the groups;

(3) Determination of the observed frequencies (O_i) specific for each grouping interval i by the relation:

$$O_i = n_i , \qquad (3)$$

where n_i is the number of values of the x parameter from the grouping interval i;

(4) Calculate the arithmetic average ($\bar{x}$), dispersion (s^2) and average square diverting (s) of the experimental data, in accordance with the specific statistical processing relations and presented in various specialized papers [1];

(5) Calculate the normal variable (u_i) for each grouping interval i, using the relationship:

$$u_i = \frac{x_{is} - \bar{x}}{s} , \qquad (4)$$

where x_{is} is the upper limit of each grouping interval, i;

(6) Determine the value of the normal distribution function (cumulative distribution function tested), $F(u_i)$ from the specific tables [1];

(7) Calculate the probabilities p_i with the relation:

$$p_i = F(u_{i+1}) - F(u_i) , \qquad (5)$$

where u_i and u_{i+1} are the limits of the i group;

(8) Calculate the expected average frequencies E_i (expected frequency) with the relation:

$$E_i = n \cdot p_i ; \qquad (6)$$

(9) The deviations between the values of the observed absolute frequencies (O_i) and the estimated average frequencies (E_i) are calculated with the relation:

$$O_i - E_i ; \qquad (7)$$

(10) Calculate the ratio $\frac{(O_i - E_i)^2}{E_i}$ for every grouping interval i;

(11) The calculated value is determined (χ^2_{calc}) specific to this test with the relation:

$$\chi^2_{calc} = \sum_{i=1}^{n_g} \frac{(O_i - E_i)^2}{E_i} ; \qquad (8)$$

(12) Determining the critical value of the Chi-Square test, $\chi^2_{crit}(\alpha; \nu)$, where
   α = significance level ($\alpha = 0.05$);
   dF = degrees of freedom, $dF = n_g - 2 - 1$, where 2 is the number of estimated parameters on the selection ($\bar{x}$ and s) and 1 is a unit; dF is determined from the specific tables [1];

(13) Comparing the calculated value (χ^2_{calc}) with the critical value (χ^2_{crit}) of the Chi-Square test, two situations can occur [1, 11]:
   a) If $\chi^2_{calc} < \chi^2_{crit}$ then the repartition hypothesis is accepted by the selection of experimental data is normal;
   b) If $\chi^2_{calc} > \chi^2_{crit}$ then the repartition hypothesis is not accepted by the selection of experimental data is normal.

4. Experimental Procedure

The studied materials, were a Ni-Cu-Cr cast iron with the following composition (% in weight): 3.47 %C; 2.14 %Si; 0.37 %Mn; 0.015 %P; 0.003 %S; 0.080 %Mg; 0.54 %Ni, 0.38 %Cu and 0.35 %Cr. The data presented in this paper are part of a comprehensive study on susceptibility to brittle thermal breakage of S.G. cast iron. For this purpose, a number of 60 Charpy U specimens, with the standard dimensions of 10×10×55 mm, were used, using a potential Charpy hammer of 300 J, according to SR EN ISO 148-1: 2016, "Metallic materials - Charpy pendulum impact test - Part I: Test method" [14].

This materials were subjected to a heat treatment whose parameters have been:
   - the austenizing temperature, $t_A = 900$ [°C];

- the maintained time at austenizing temperature, τ_A = 30 [min];
- the temperature at isothermal level, t_{iz} = 300 and 400 [°C];
- the maintained time at the isothermal level, τ_{iz} = 60 [min], corresponding to the first stage for obtaining bainitic structure.

All these experimental specimens were performed at isothermal maintenance in salt-bath (55% KNO3+45% NaNO3), being the cooling after the isothermal maintenance was done in air.

From this material, 60 typical test uncreated specimens for impact strength (KC) determination was done (30 specimens for each temperature level at isothermal maintenance).

5. Applying the Chi-Square Test

For the ease of applying the Chi test and for statistical processing, the results were divided into a number of groups n_g = 10, for each study group, the boundaries of the intervals were determined, and later they were delimited: the arithmetic mean ($\bar{x}$ = 40.48), average square deviation (s = 5.22) and the observed absolute frequency (O_i). In order to determine estimated average frequencies E_i (expected frequency), the specific calculations presented in the steps are performed (7)-(9), the values of the normal variable were first calculated (u_i), taking into account the boundaries of each grouping interval [1, 10]:

$$u_1 = \frac{32 - 40.48}{5.22} = -1.62, \qquad u_2 = \frac{34 - 40.48}{5.22} = -1.24, \qquad u_3 = \frac{36 - 40.48}{5.22} = -0.85,$$

$$u_4 = \frac{38 - 40.48}{5.22} = -0.47, \qquad u_5 = \frac{40 - 40.48}{5.22} = -0.09, \qquad u_6 = \frac{42 - 40.48}{5.22} = 0.29,$$

$$u_7 = \frac{44 - 40.48}{5.22} = 0.67, \qquad u_8 = \frac{46 - 40.48}{5.22} = 1.05, \ u_9 = \frac{48 - 40.48}{5.22} = 1.44,$$

$$u_{10} = \frac{50 - 40.48}{5.22} = 1.82.$$

With the help of these results and taking into account the value of the normal distribution function $F(u_i)$ from the specific tables [1], the values of the estimated average frequencies E_i (expected frequency) were determined by means of the relations presented in step (10):

$E_1 = 60 \cdot [F(u_1)] = 60 \cdot [0.0465] = 2.79;$
$E_2 = 60 \cdot [F(u_2) - F(u_1)] = 60 \cdot [F(-1.24) - F(-1.62)] = 60 \cdot [0.1057 - 0.0465] = 3.552;$
$E_3 = 60 \cdot [F(u_3) - F(u_2)] = 60 \cdot [F(-0.85) - F(-1.24)] = 60 \cdot [0.2005 - 0.1057] = 5.688;$
$E_4 = 60 \cdot [F(u_4) - F(u_3)] = 60 \cdot [F(-0.47) - F(-0.85)] = 60 \cdot [0.3372 - 0.2005] = 8.202;$
$E_5 = 60 \cdot [F(u_5) - F(u_4)] = 60 \cdot [F(-0.09) - F(-0.47)] = 60 \cdot [0.5000 - 0.3372] = 9.768;$
$E_6 = 60 \cdot [F(u_6) - F(u_5)] = 60 \cdot [F(0.29) - F(-0.09)] = 60 \cdot [0.6141 - 0.5000] = 6.846;$
$E_7 = 60 \cdot [F(u_7) - F(u_6)] = 60 \cdot [F(0.67) - F(0.29)] = 60 \cdot [0.7486 - 0.6141] = 8.07;$
$E_8 = 60 \cdot [F(u_8) - F(u_7)] = 60 \cdot [F(1.05) - F(0.67)] = 60 \cdot [0.8531 - 0.7486] = 6.27;$
$E_9 = 60 \cdot [F(u_9) - F(u_8)] = 60 \cdot [F(1.44) - F(1.05)] = 60 \cdot [0.9251 - 0.8531] = 4.32;$
$E_{10} = 60 \cdot [F(u_{10}) - F(u_9)] = 60 \cdot [F(1.82) - F(1.44)] = 60 \cdot [0.9649 - 0.9251] = 2.388.$

The statistical resolution of the test in accordance with steps (10) - (13) and the general presentation of the data used is shown in Table 1.

In order to verify the hypothesis that the distribution of the experimental data analyzed follows a normal distribution, the critical value of the Chi-Square test, $\chi^2_{crit}(0.05; 7)$ = 14.067 [1] was established.

Comparing the calculated value, $\chi^2_{calc}(8.0154)$ with the critical value, $\chi^2_{crit}(14.067)$ of the Chi-Square test, it follows that the condition: $\chi^2_{calc} < \chi^2_{crit}$ is satisfied, so the assumption that the distribution of the experimental data analyzed follows a normal distribution.

Figure 1 shows the correspondence between the distribution of absolute frequencies observed (O_i) and the number of statistical groups.

Tablel 1. The statistical resolution of the test

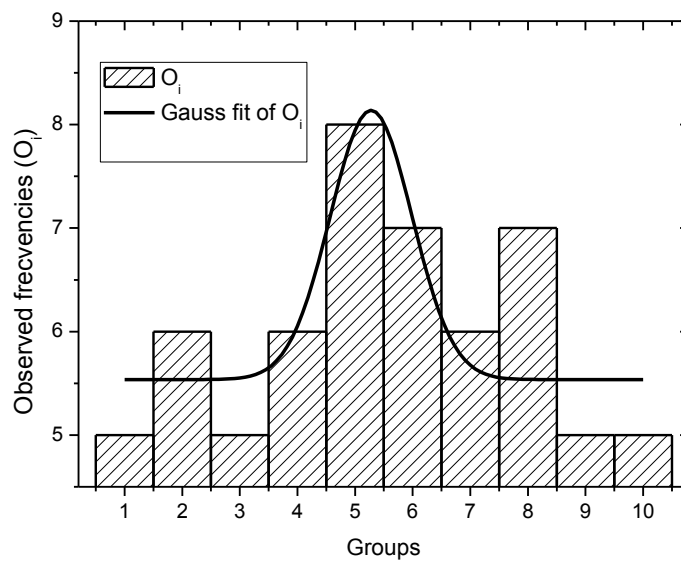| n_g	Grouping interval		O_i	E_i	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
	x_{min}	x_{max}				
1	30	32	5	2.79	4.8841	1.7506
2	32	34	6	3.552	5.9927	1.6871
3	34	36	5	5.688	0.4733	0.0832
4	36	38	6	8.202	4.8488	0.5912
5	38	40	8	9.768	3.1258	0.3200
6	40	42	7	6.846	0.0237	0.0035
7	42	44	6	8.07	4.2849	0.5309
8	44	46	7	6.27	0.5329	0.0849
9	46	48	5	4.32	0.4624	0.1070
10	48	50	5	2.388	6.8225	2.8570
Σ	-	-	60	-	-	8.0154

Fig. 1. Correspondence between the distribution of the absolute frequencies observed (O_i) and the number of statistical groups (n_g)

Analyzing the results presented in Table 1 and Figure 1, it is noted that the assumption that the distribution of the experimental data analyzed follows a normal distribution and by overlapping the graphical representation of the data of absolute frequencies observed (O_i), it is observed that they describe appropriate form of a normal distribution frequency function (Gauss).

6. Conclusion
Analyzing all data taken into account there can be said the following:
(a) using the Chi-Square adjustment test in this paper, check that the (experimental) selection distribution falls within a certain class of theoretical distribution;
(b) the data presented in this paper are part of a comprehensive study on susceptibility to brittle fracture of a thermally treated Ni-Cu-alloyed S.G. cast iron;
(c) the Chi-Square test was performed following the 12 specific calculation steps;
(d) by comparing the calculated value, χ^2_{calc} (8.0154) with the critical value, χ^2_{crit} (14.067) of the Chi-Square test, it follows that the condition: $\chi^2_{calc} < \chi^2_{crit}$ is satisfied, therefore assuming that the distribution of the experimental data analyzed follows a normal distribution.

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